N-Particle Scattering and Asymptotic Completeness in Interacting Wedge-local QFT Models

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Mathematics of interacting QFT models, York, July 1-5, 2019

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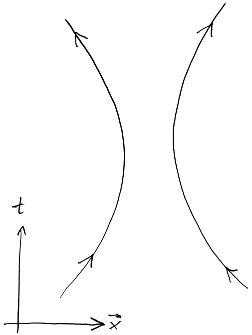

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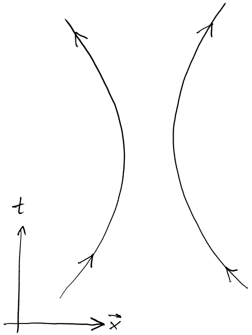

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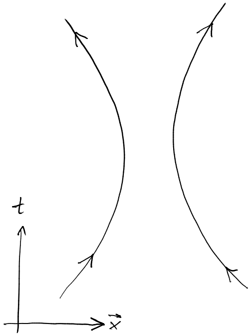

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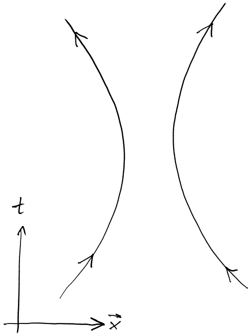

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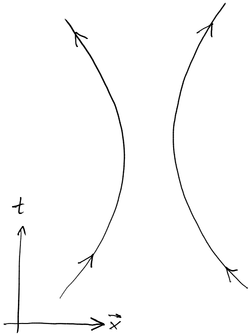

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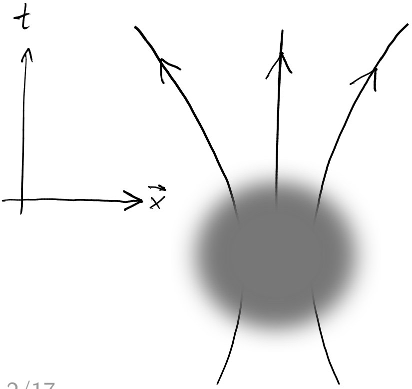
More recent progress: Rigorous constructions of "almost" QFTs ("**wedge-local**") exhibiting non-trivial 2-particle interactions. [Grosse, Lechner'07] [Buchholz, Lechner, Summers'11]

What is the physical interpretation of these models?

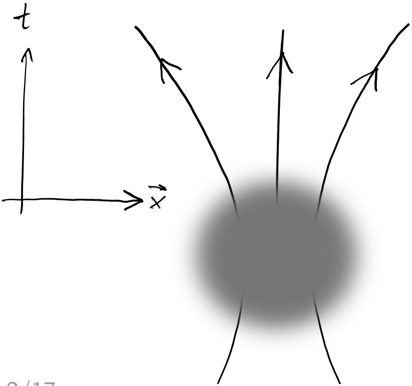
Problematic Geometry of Wedge-local 3-Particle Scattering



Problematic Geometry of Wedge-local 3-Particle Scattering

- Scattering Amplitudes

$$S_{fi} = {}^{\text{out}}\langle 1\,2\,3 | 1'\,2' \rangle^{\text{in}}$$



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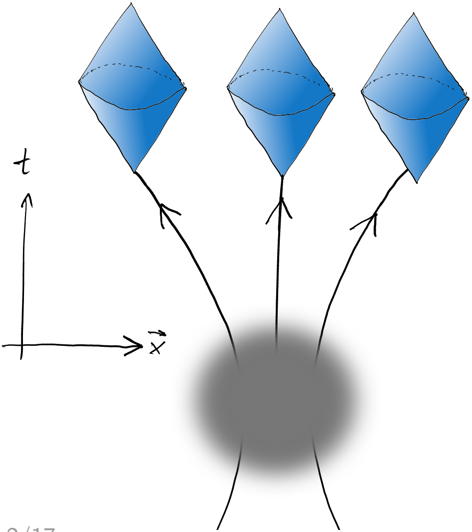

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$$|1\,2\,3\rangle^{\text{out}} := \lim_{\tau\to\infty} B_{1\tau} B_{2\tau} B_{3\tau}|\Omega\rangle$$

$$B_{k\tau}|\Omega\rangle \underset{\tau\to\infty}{\to} |k\rangle$$

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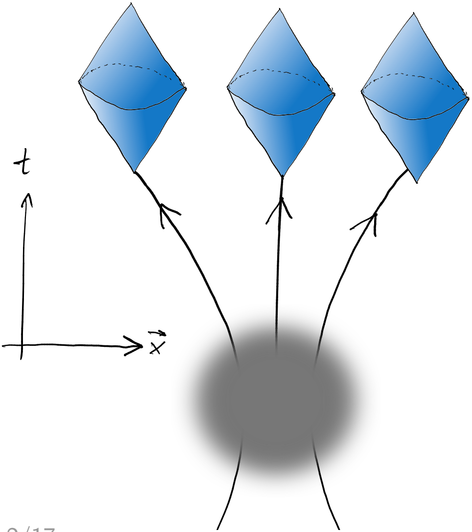

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- Existence of Limit proven using **Separation of Localizations**

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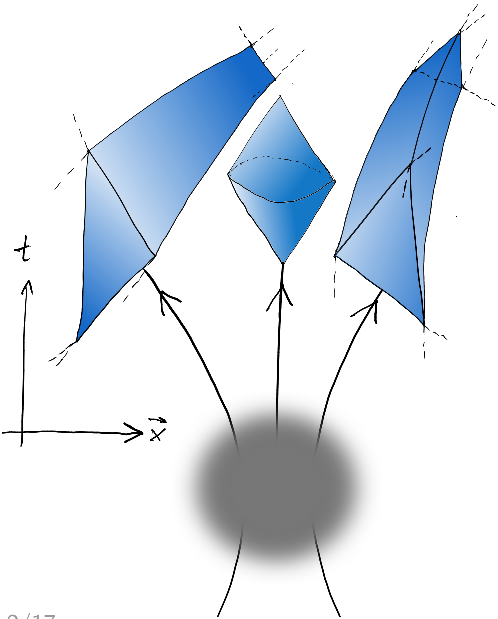

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- Wedges $\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3$ cannot pairwise space-like separate!

Overview

What is a Wedge-local QFT?

- Field Theory: $\phi(x)$ measurable quantity associated to space-time point $x \in \mathbb{R}^{s+1}$ (e.g. electromagn. fields)

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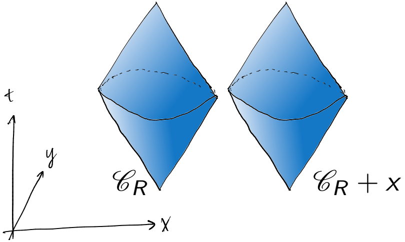
- Local QFT: $\phi(x)$ localized in $\mathscr{C}_R + x$, $x = (t, \mathbf{x}) \in \mathbb{R}^{s+1}$:

$$\mathscr{C}_R + x_1,\ \mathscr{C}_R + x_2 \text{ space-like} \implies [\phi(x_1), \phi(x_2)] = 0$$



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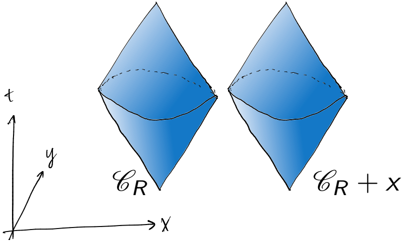
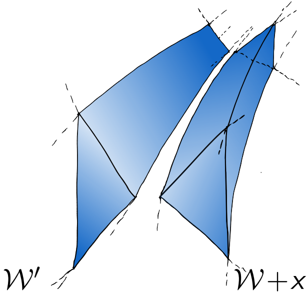


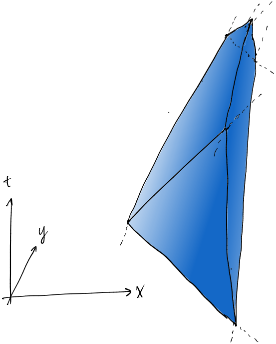


- Wedge-Local QFT: $\phi_{\mathcal{W}}(t, \mathbf{x})$ localized in $\mathcal{W} + (t, \mathbf{x})$:

$$x_1 + \mathcal{W}_1,\ x_2 + \mathcal{W}_2 \text{ space-like} \implies [\phi_{\mathcal{W}_1}(x_1), \phi_{\mathcal{W}_2}(x_2)] = 0$$

MATHEMATICALLY/PHYSICALLY WEAKER!

Family of Rindler-Wedge-Regions in Space-Time



Rindler Reference Wedge:

$$\mathcal{W}_\mathrm{r} := \{(t, \mathbf{x}) \in \mathbb{R}^{s+1} : |t| < x_1\}$$

Definition: General Wedge regions $\mathcal{W}$ are generated by Poincaré transformations $\lambda \in \mathcal{P} = \mathcal{L} \ltimes \mathbb{R}^{s+1}$

$$\mathcal{W} = \lambda \mathcal{W}_\mathrm{r} = \Lambda \mathcal{W}_\mathrm{r} + x$$

Elementary advantages: Highly symmetric, causally closed, ...

Axiomatic framework for Wedge-local QFT

Wedge-local model defined by specifying the following mathematical objects $(\mathfrak{A}, \alpha, \Omega, \mathscr{H})$.

- Hilbert space $\mathscr{H}$ of vector states
- Distinguished *vacuum* state $\Omega \in \mathscr{H}$
- "Net" of von Neumann algebras $\mathcal{W} \mapsto \mathfrak{A}(\mathcal{W}) \subset \mathrm{B}(\mathscr{H})$, $\mathcal{W} \subset \mathbb{R}^{s+1}$ wedge region in space-time
- Space-time translations of states $(t, \mathbf{x}) \mapsto U(t, \mathbf{x}) = e^{\mathrm{i}tH - \mathrm{i}\mathbf{x}\cdot\boldsymbol{P}}$
- Translations of observables $\alpha_x A := A(x) := U(x)\, A\, U(x)^*$

These objects $(\mathfrak{A}, \alpha, \Omega, \mathscr{H})$ further have to satisfy the wedge-local **Haag-Kastler postulates**.

Firstly, minimal assumptions required for a sensible interpretation of $A \in \mathfrak{A}(\mathcal{W}) \subset \mathrm{B}(\mathscr{H})$ "being **localizable**" in wedge $\mathcal{W} \subset \mathbb{R}^{s+1}$,

(HK1) **Isotony:** $\mathcal{W}_1 \subset \mathcal{W}_2 \Longrightarrow \mathfrak{A}(\mathcal{W}_1) \subset \mathfrak{A}(\mathcal{W}_2)$

(HK2) **Wedge-Locality:** $\mathcal{W}_1 \subset \mathcal{W}_2' \Longrightarrow \mathfrak{A}(\mathcal{W}_1) \subset \mathfrak{A}(\mathcal{W}_2)'$

(HK3) **Translation-Covariance:** $\alpha_x \mathfrak{A}(\mathcal{W}) = \mathfrak{A}(\mathcal{W} + x)$

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Secondly, need assumptions on structure of Hilbert space of states:

(HK4) **Uniqueness of the vacuum Ω**

(HK5) **Haag-Ruelle Spectrum Condition**:

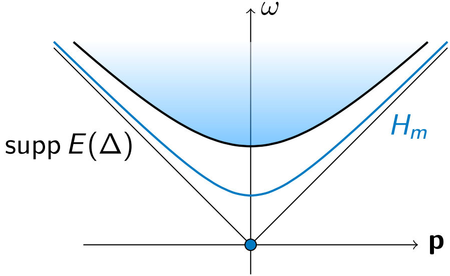
- Positivity of Energy
- Existence of Isolated Mass Shell
  (Stable 1-particle states, purely massive theory)

(HK6) **Cyclicity of Ω**

Analysis of the Particle Content & Spectrum Condition

Space-time translations α *unitarily implemented*: $(A \in \mathfrak{A},\ x = (t, \mathbf{x}))$

$$A(x) := \alpha_x(A) = U(x)AU(x)^*$$



Generators of space-time translations:

$$U(t, \mathbf{x}) = \mathrm{e}^{\mathrm{i}Ht - \mathrm{i}\boldsymbol{P}\cdot\mathbf{x}}$$

Shape of joint spectrum of $(H, \boldsymbol{P})$ specified by spectrum condition:

$$\sigma_{(H,\boldsymbol{P})} = \{0\} \cup H_m \cup \bar{H}_{2m}$$

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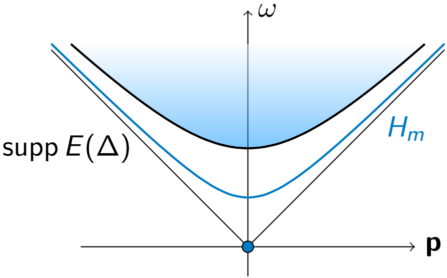

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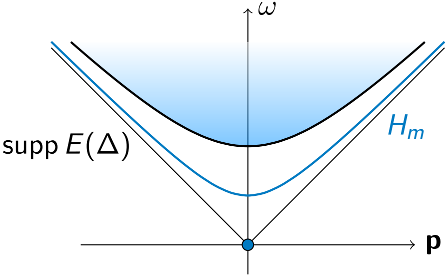

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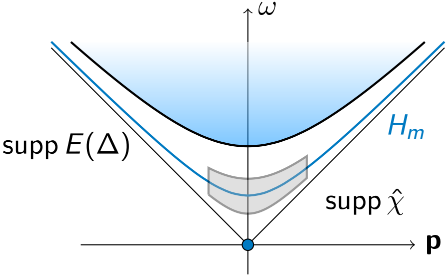

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Mass Gaps $\Rightarrow$ Separation of H_m and $\sigma_{(H,\boldsymbol{P})} \setminus H_m$ via $\hat{\chi} \in \mathscr{S}(\mathbb{R}^{s+1})$

Definition of Haag-Ruelle Creation-Op. Approximants

From a given wedge-local operator $A \in \mathfrak{A}(\mathcal{W})$ can construct new operators by space-time translations $\alpha_x(A)$ and via superpositions.

Combined: **Space-time Smearing** of A with $\chi : \mathbb{R}^{s+1} \longrightarrow \mathbb{C}$,

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Apply: Construct Solution of 1-Particle Problem [Haag, Ruelle'60s]

(Step 1) **Construction of 1-Particle States**
If $\hat{\chi}$ separates mass shell from remaining spectrum, $B = A(\chi)$ creates 1-particle states from vacuum:

$$B\Omega \in \mathscr{H}_1 = E(H_m)\mathscr{H}$$

(Step 2) **Introduce Comparison Dynamics**
Adding spatial smearing with Klein-Gordon solution f
$\Longrightarrow$ τ-independent one-particle vector $B_\tau(f)\Omega$,
created at time τ.

 But: Wedge-Localization is obstacle for multi-particle problem!

Towards Construction of N-Particle Scattering States

Important: Localization and Ordering of Wave Packets and B_τ's

$$f(t,\mathbf{x}) := \int \mathrm{d}^s k\ \mathrm{e}^{-\mathrm{i}\omega_m(\mathbf{k})t+\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\ \tilde{f}(\mathbf{k}), \qquad \omega_m(\mathbf{k}) := \sqrt{\mathbf{k}^2+m^2},$$

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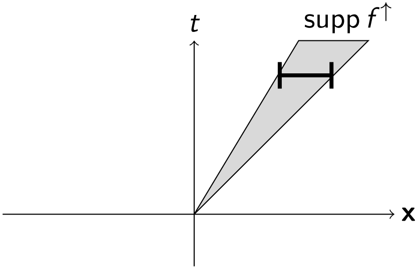

Defs.: *Velocity support*:

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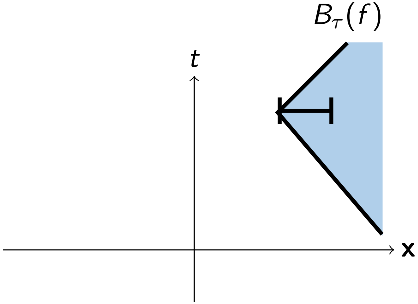

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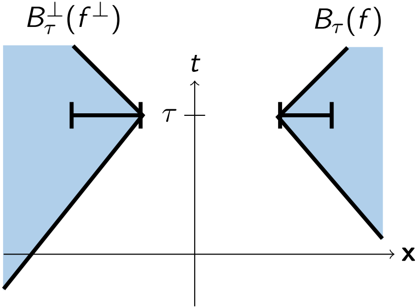

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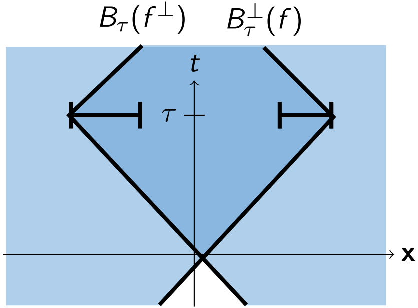

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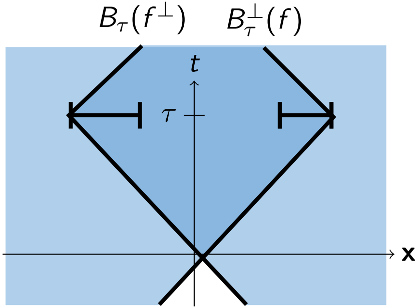

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Precursor Order Relation:

$$\mathcal{V}_1 \prec_{\mathcal{W}} \mathcal{V}_2 :\Leftrightarrow \mathcal{V}_2 - \mathcal{V}_1 \subset \mathcal{W}.$$
$$(\mathcal{V}_k \subset \mathbb{R}^{s+1}, \ \mathcal{W} \text{ centered})$$

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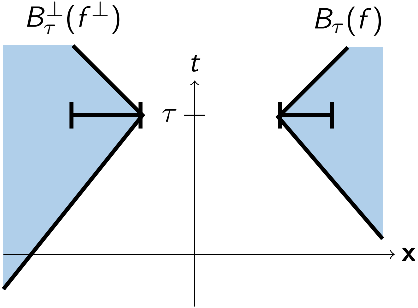

Defs.: *Velocity support*:

$$\mathcal{V}(f) := \{(1, \tfrac{\mathbf{k}}{\omega_m(\mathbf{k})}) : \mathbf{k} \in \operatorname{supp} \tilde{f}\}$$

Precursor Order Relation:

$$\mathcal{V}_1 \prec_{\mathcal{W}} \mathcal{V}_2 :\Leftrightarrow \mathcal{V}_2 - \mathcal{V}_1 \subset \mathcal{W}.$$
$$(\mathcal{V}_k \subset \mathbb{R}^{s+1},\ \mathcal{W} \text{ centered})$$

Proposition: Correct Ordering leads to Commutator Decay.

Construction of N-Particle Scattering States [MD'18]

Ingredient (1): **Correct Ordering**

Let $A_k \in \mathfrak{A}(\mathcal{W})$, $(1 \leq k \leq n)$, $B_k := A_k(\chi)$, and f_k s.t.

$$\mathcal{V}(f_n) \prec_{\mathcal{W}} \mathcal{V}(f_{n-1}) \prec_{\mathcal{W}} \ldots \prec_{\mathcal{W}} \mathcal{V}(f_1).$$

Then corresponding outgoing scattering state defined by

$$\Psi^+ := \lim_{\tau \to \infty} B_{1\tau}(f_1) B_{2\tau}(f_2) \ldots B_{n\tau}(f_n)\Omega,$$

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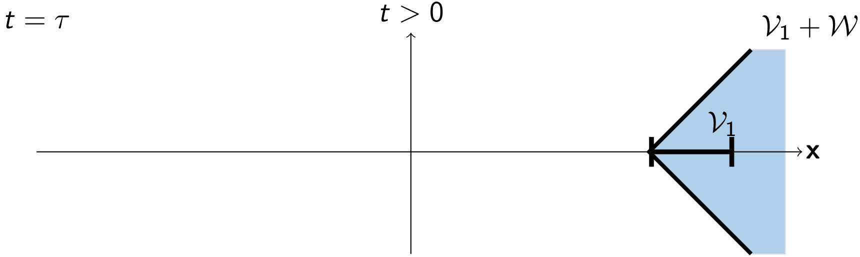

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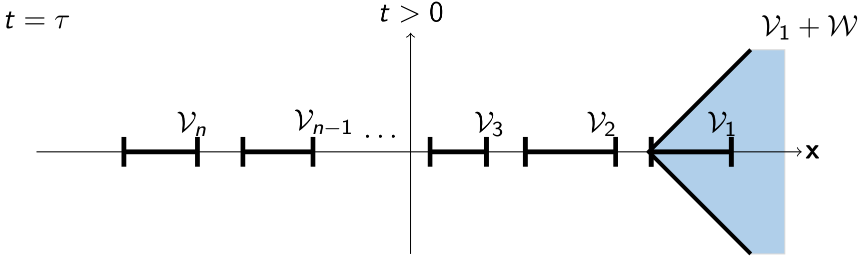

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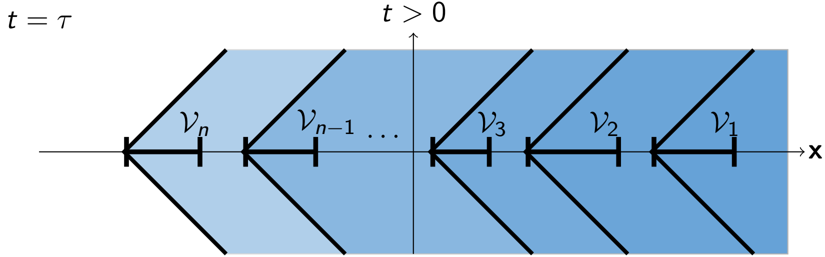

(2) Wedge-Swapping Symmetry of 1-Particle States

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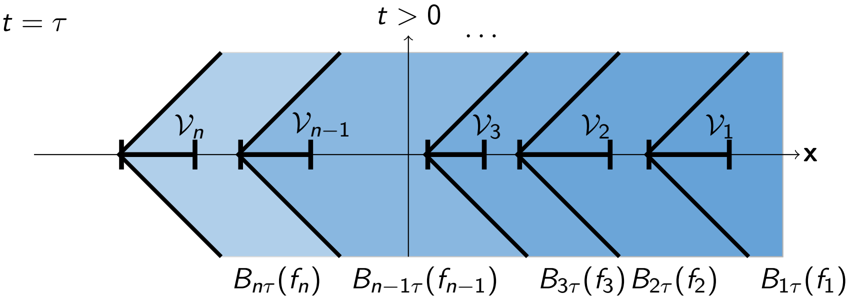

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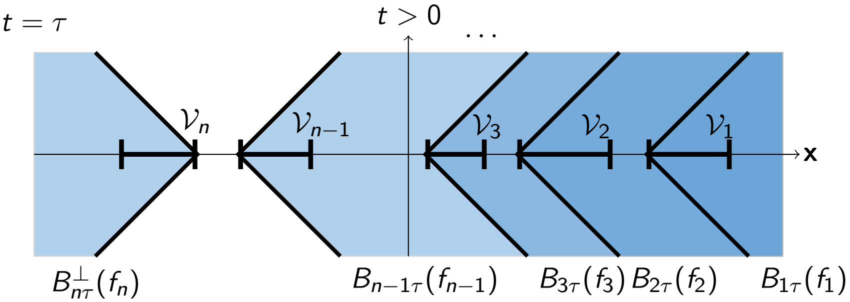

Def.: A one-particle state $\Psi_1 \in \mathscr{H}_1$ is **swappable** w.r.t. $\mathcal{W}$ if

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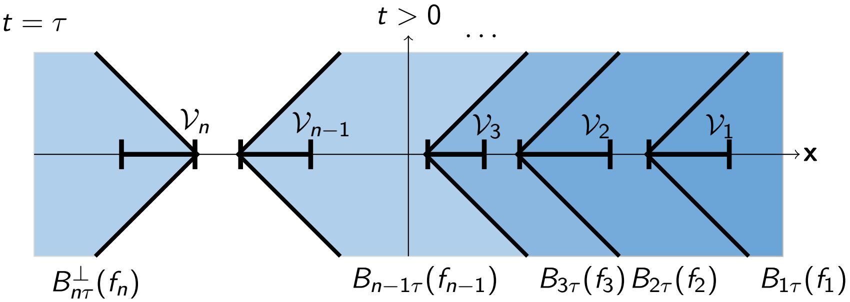

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Remark: Swappable Ψ_1 can be constructed from **Wedge duality** $\mathfrak{A}(\mathcal{W})' = \mathfrak{A}(\mathcal{W}')$ using Tomita-Takesaki Theory, dense in $\mathscr{H}_1$.

Main Result: Wedge-local Haag-Ruelle Theorem

Fix a wedge $\mathcal{W}$, let $\Psi_k = E(H_m)A_k\Omega = E(H_m)A_k^{\perp}\Omega$ **swappable**, i.e. $A_k \in \mathfrak{A}(\mathcal{W})$, $A_k^{\perp} \in \mathfrak{A}(\mathcal{W}^{\perp})$, and assume isolated mass shells.

Let $f_1, \ldots, f_n$ regular Klein-Gordon solutions with **velocities** $\mathcal{V}(f_k)$ **ordered** s.t.

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Theorem. [MD'18] (1) $\Psi^+ := \lim_{\tau \to +\infty} \Psi(\tau)$ convergent.

(2) For fixed $\mathcal{W}$ with "upright geometry", scalar products of any two such Ψ^+, Ψ'^+ are given by the Fock structure relation

$$\langle \Psi^+, \Psi'^+ \rangle = \delta_{nn'} \prod_{k=1}^{n} \langle \Psi_k, \Psi'_k \rangle .$$

Interpretation: Ψ^+ outgoing scattering state

Remark: get also incoming Ψ^-, but need **opposite ordering**

Proof Idea (Convergence of 3-Particle Out-States)

$A_k \in \mathfrak{A}(\mathcal{W})$, $A_k^\perp \in \mathfrak{A}(\mathcal{W}^\perp)$, s.t. $E_m A_k \Omega = E_m A_k^\perp \Omega$, $(1 \leq k \leq 3)$
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Proof of Convergence via Cook's Method: For $\tau_2 > \tau_1 > 0$,

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Recall: $\mathcal{V}_{f_3} \prec \mathcal{V}_{f_2} \prec \mathcal{V}_{f_1}$

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Answer: Excluded by (Fock structure) result [MD'18].

Ordered Asymptotic Completeness in Wedge-local QFT

Application: Asymptotic Completeness, GL-Models

- Wedge-local Møller-Operators $\mathbf{W}^{\pm}_{\mathcal{W}}$ can exhibit dependence on the preparation wedge $\mathcal{W}$ (ruled out in local QFT),

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Def.: A wedge-local QFT $(\mathfrak{A}, U, \Omega, \mathscr{H})$ is **asymptotically complete (AC)**, if $\overline{\mathbf{W}^{\pm}_{\mathcal{W}} \mathcal{V}^{\succ_{\mathcal{W}}/\prec_{\mathcal{W}}}} = \mathscr{H}$ for any wedge region $\mathcal{W}$.

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Def.: A wedge-local QFT $(\mathfrak{A}, U, \Omega, \mathscr{H})$ is **asymptotically complete (AC)**, if $\overline{\mathbf{W}^{\pm}_{\mathcal{W}} \mathcal{V}^{\succ_{\mathcal{W}} / \prec_{\mathcal{W}}}} = \mathscr{H}$ for any wedge region $\mathcal{W}$.

Lemma. (in preparation) In the models of [Grosse,Lechner'07] and [Buchholz,Lechner,Summers'11] we have $\mathbf{W}^{\pm}_{Q,\mathcal{W}} = \mathbf{W}^{\pm}_{0,\mathcal{W}} S_Q^{\succ_{\mathcal{W}} / \prec_{\mathcal{W}}}$, with unitary
$$S_Q^{\succ_{\mathcal{W}} / \prec_{\mathcal{W}}} = \prod_{1 \le i < j \le N} \mathrm{e}^{\mathrm{i} P_i \cdot Q P_j / 2}, \quad Q \text{ GL deformation matrix.} \qquad (\star)$$

Application: Asymptotic Completeness, GL-Models

- Wedge-local Møller-Operators $\mathbf{W}^{\pm}_{\mathcal{W}}$ can exhibit dependence on the preparation wedge $\mathcal{W}$ (ruled out in local QFT),

$$\mathbf{W}^{\pm}_{\mathcal{W}} \Psi_1 \otimes \ldots \otimes \Psi_N := \lim_{\tau \to \pm\infty} B_{1\tau}(f_1) \ldots B_{N\tau}(f_N)\Omega,$$

where $B_{k\tau}(f_k)\Omega = \Psi_k$, $B_k = A_k(\chi)$, $A_k \in \mathfrak{A}(\mathcal{W})$.

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Kor. BLS-deformed model AC $\Longleftrightarrow$ underlying undeformed model AC.

Thm. N-particle states of GL-model have factorizing scattering data $(\star)$. Hence the GL-Model is interacting and asymptotically complete

Example: Failure of Asymptotic Completeness

Inspiration: [Longo,Tanimoto,Ueda'17] [D'Antoni,Longo,Rădulescu'01]

$\mathscr{H}_1 := L^2(\mathbb{R}, \mathrm{d}\theta)$

$\mathscr{H} := \Gamma^u(\mathscr{H}_1) = \bigoplus_{k=0}^{\infty} \bigotimes^k \mathscr{H}_1$ (unsymmetrized)

$U(x, \Lambda) = \Gamma(U_1(x, \Lambda))$

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Define fields ($f \in \mathscr{S}(\mathbb{R}^2)$, $m > 0$)

$$\Phi(f) := z^*(f^+) + z(f^-), \quad \Phi'(f) := J\Phi(f^*)J,$$

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Observation: ordered incoming and outgoing states are orthogonal, ordered AC fails.

Outlook and Summary

- Scattering Theory of Haag and Ruelle has been extended to massive wedge-local theories [MD'18]. Most notably, a fully general treatment of the $N \geq 3$-particle case is provided.
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Thank you for your attention.