# The arrow of time and quantum physics: difficulties and resolutions

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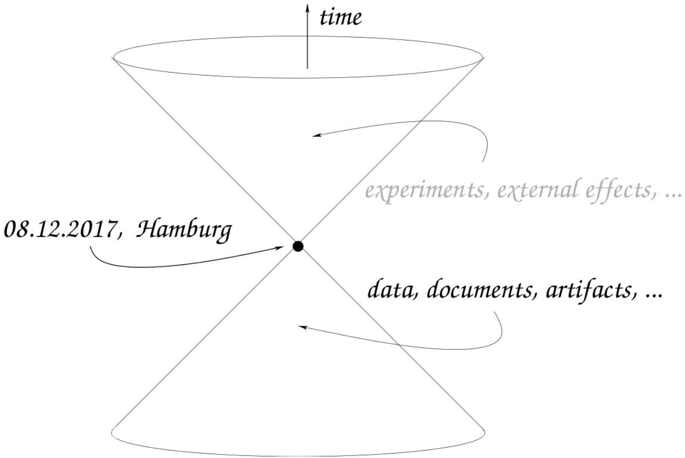
Quantum physics meets mathematics
Syposium on the occasion of **Klaus Fredenhagen**'s 70th birthday
Universität Hamburg, December 8th 2017

# Arrow of time



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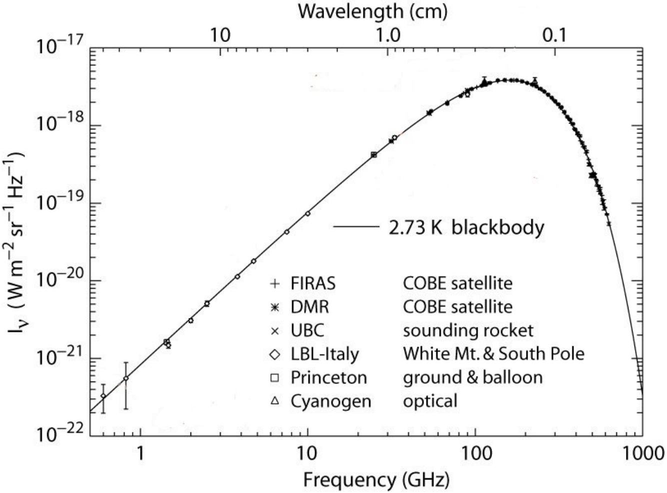

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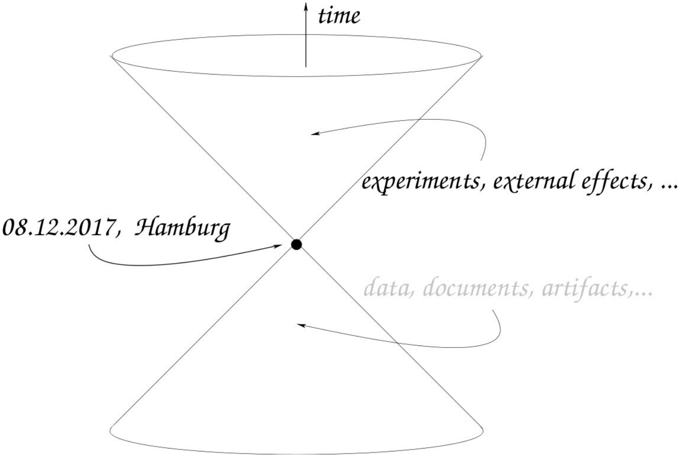

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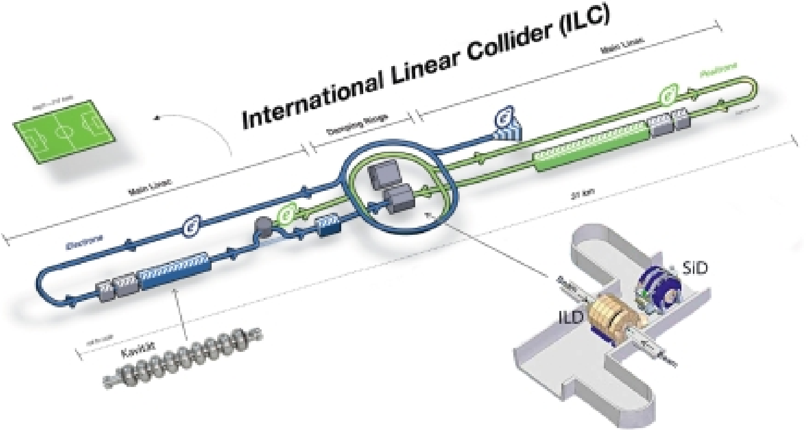

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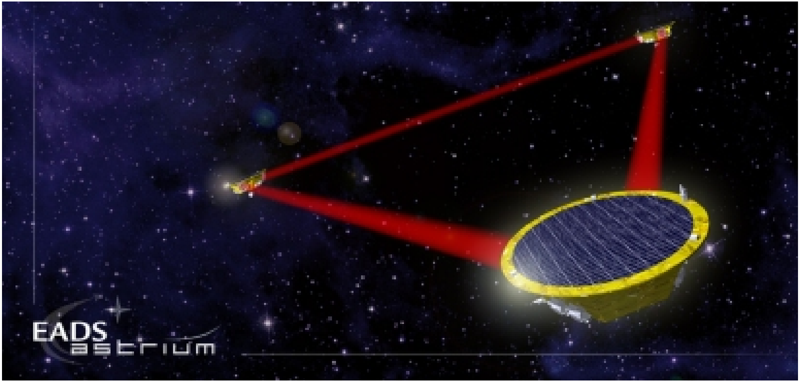

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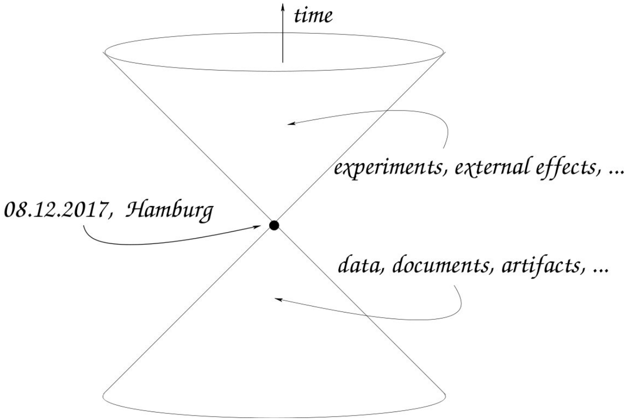
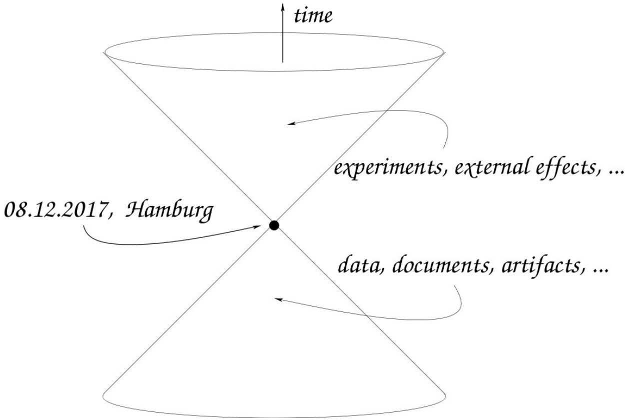



Only parts of Minkowski space (forward lightcones) are accessible

# Arrow of time




Physical time evolution (inertial observer) acts as a *semi-group*

# Quantum physics

**Basic conepts**

Observables: $\mathcal{A}$ unital algebra of bounded operators in some cone
Arrow of time: time evolution (inertial observer) acts by morphisms

$$\alpha_t(\mathcal{A}) \subset \mathcal{A}\,, \quad t \in \mathbb{R}_+$$

States: expectation functionals in $\mathcal{A}^*$. Preceeding structure suffices to characterize ground states $\omega$ (invariance, analyticity, mixing)

**Facts**

*Let $\omega$ be a ground state on $(\mathcal{A}, \alpha)$ with GNS representation $(\pi, \mathcal{H}, \Omega)$.*

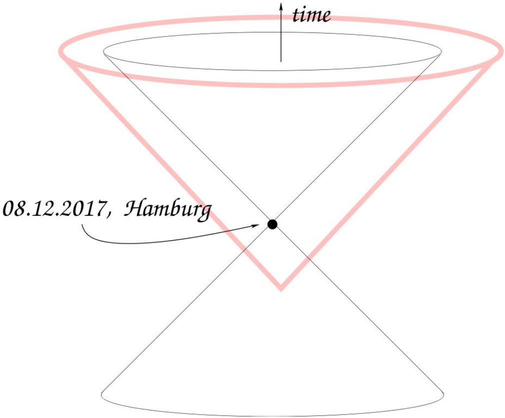
1. *There is a continuous unitary representation $U$ of $\mathbb{R}$ with positive generator s.t. $\mathrm{Ad}\, U(t) \circ \pi = \pi \circ \alpha_t$, $t \in \mathbb{R}_+$, and $U(t)\Omega = \Omega$, $t \in \mathbb{R}$.*
2. *There are the alternatives: (i) $\pi(\mathcal{A})'' = \mathcal{B}(\mathcal{H})$ (massive theories) (ii) $\pi(\mathcal{A})''$ type $III_1$ (presence of massless particles)*

## Interpretation

*Let $\omega$ be a ground state on $\mathcal{A}$ with GNS representation $(\pi, \mathcal{H}, \Omega)$.*

1. *The unitary representation U (fixed by theory) allows to extend the state $\omega$ to the past, from the data taken in any given future directed lightcone. (Justification of treatment of time as $\mathbb{R}$).*
2. *In massive theories these date uniquely determine this extension.*

   *In presence of massless particles the extension is* ***not unique****, leading to conceptual problems.*




Incomplete information about the past (outgoing radiation)

*Fiat lux!*

Implications: Standard theoretical concepts of quantum physics become operationally irrelevant

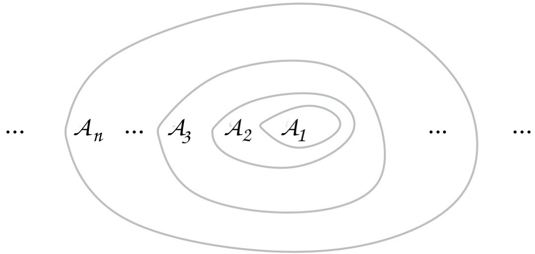
- pure states? : incomplete information!
- superposition principle? : no lifts to rays in a Hilbert space!
- transition probabilities? : no minimal projections!

Are there other theoretical concepts describing the same physics?
Proposal (DB, Erling Størmer):

- *funnels* of algebras: provide locally complete information
- *generic* states: can be superimposed
- *primitive* observables: replace minimal projections

# Funnels

Observations and operations are made in (fuzzy) spacetime regions




Algebra of observables generated by

- $\mathcal{A}_1 \subset \mathcal{A}_2 \subset \cdots \subset \mathcal{A}_n \cdots$ factors of type $\mathrm{I}_\infty \simeq \mathcal{B}(\mathcal{H})$
- $\mathcal{A}_n' \bigcap \mathcal{A}_{n+1}$ infinite dimensional (hence type $\mathrm{I}_\infty$), $n \in \mathbb{N}$
- $\boldsymbol{\mathcal{A}} = \bigcup_n \mathcal{A}_n$ proper sequential type $\mathrm{I}_\infty$ funnel (Takesaki)

Examples: relativistic QFTs (split property), lattice theories, ...

# Generic states

States $\omega : \boldsymbol{\mathcal{A}} \to \mathbb{C}$, GNS–representation $(\pi, \mathcal{H}, \Omega)$

- locally normal, *i.e.* weakly continuous on unit balls of $\mathcal{A}_n$, $n \in \mathbb{N}$,
- faithful, *i.e.* $\omega(A^*A) = 0$ for $A \in \boldsymbol{\mathcal{A}}$ implies $A = 0$
- generic, *i.e.* representing vector $\Omega$ cyclic for $\mathcal{A}_n' \bigcap \mathcal{A}_{n+1}$, $n \in \mathbb{N}$

Remark: Generic vector states "$G_\delta$ dense" in $\mathcal{H}_1$ (Dixmier, Marechal)

## Definition

Let $\omega$ be generic. Its orbit under non-mixing operations is given by

$$\boldsymbol{\omega_{\mathcal{A}}} \doteq \{\omega_A = \omega \circ \mathrm{Ad}\, A \ : \ A \in \boldsymbol{\mathcal{A}}, \ \omega_A(1) = 1\},$$

where $\mathrm{Ad}\, A\,(B) = A^* B\, A, \quad B \in \boldsymbol{\mathcal{A}}$.

**Physical interpretation:**

Generic states $\omega$ describe a "global background" in which physical operations are performed ("state of the world"). Given such a state, these operations produce the corresponding orbit $\boldsymbol{\omega_{\mathcal{A}}}$.

**Examples:**

- vacuum states in relativistic QFT
- thermal equilibrium states in relativistic and non-relativistic QFT
- Hadamard states in curved spacetimes

# Superpositions

Fix a generic state $\omega$ with orbit $\boldsymbol{\omega}_{\boldsymbol{\mathcal{A}}}$. Norm distance of states

$$\|\omega_A - \omega_B\| \doteq \sup_{C \in \boldsymbol{\mathcal{A}}_1} |\omega_A(C) - \omega_B(C)| \, , \quad \omega_A, \omega_B \in \boldsymbol{\omega}_{\boldsymbol{\mathcal{A}}} \, .$$

## Proposition

*There exists a canonical lift from* $\boldsymbol{\omega}_{\boldsymbol{\mathcal{A}}}$ *to rays in* $\boldsymbol{\mathcal{A}}$ *which is*

1. *bijective:* $\omega_A = \omega_B$ *iff* $B = t\,A$ *for* $t \in \mathbb{T}$
2. *locally continuous: if* $\|\omega_{A_m} - \omega_A\| \to 0$ *for (bounded)* $A_m, A \in \mathcal{A}_{\boldsymbol{n}}$*, then* $t_m A_m \to A$ *in the strong operator topology*
3. *locally complete: if* $\|\omega_{A_l} - \omega_{A_m}\| \to 0$ *for (bounded)* $A_l, A_m \in \mathcal{A}_{\boldsymbol{n}}$*, there is* $A \in \mathcal{A}_{\boldsymbol{n}}$ *such that* $t_m A_m \to A$ *and* $\|\omega_{A_m} - \omega_A\| \to 0$.

**Physical interpretation:**

1. superposition of states in $\boldsymbol{\omega}_{\mathcal{A}}$ is a meaningful operation,
$$\omega_A, \omega_B \leftrightarrow \mathbb{T}\,A,\ \mathbb{T}\,B \rightarrow \mathbb{T}\,(c_A\,A + c_B\,B) \leftrightarrow \omega_{(c_A\,A + c_B\,B)}$$
relative phase between $c_A, c_B \in \mathbb{C}$ matters

3. $\boldsymbol{\omega}_{\mathcal{A}}$ maximal set reached by localized non-mixing operations

**Mixtures:**

$$\mathrm{Conv}\,\boldsymbol{\omega}_{\mathcal{A}} \doteq \left\{ \textstyle\sum_m p_m\,\omega_{A_m} :\ \omega_{A_m} \in \boldsymbol{\omega}_{\mathcal{A}},\ p_m > 0,\ \textstyle\sum_m p_m = 1 \right\}$$

**Proposition**

*Let* $\omega_A \in \omega_{\mathcal{A}}$ *s.t.* $\omega_A = \sum_{m=1}^{M} p_m\,\omega_{A_m}$; *then* $\omega_{A_1} = \cdots = \omega_{A_M} = \omega_A$.

$\boldsymbol{\omega}_{\mathcal{A}}$ extreme points of $\mathrm{Conv}\,\boldsymbol{\omega}_{\mathcal{A}}$; analogue of pure states.

# Transition probabilities

## Definition

Let $\omega_A, \omega_B \in \boldsymbol{\omega}_{\mathcal{A}}$. Transition probability given by: $\omega_A \cdot \omega_B \doteq |\omega(A^* B)|^2$

(Defintion meaningful in view of the bijective relations $\omega_A \leftrightarrow \mathbb{T}A$, $\omega_B \leftrightarrow \mathbb{T}B$)

Remark: comparison with Uhlmann transition probability

$$\omega_A \cdot \omega_B \leq \omega_A \overset{U}{\cdot} \omega_B = \sup_{\Omega_A, \Omega_B} |\langle \Omega_A, \Omega_B \rangle|^2.$$

## Proposition

*Let* $\omega_A, \omega_B \in \boldsymbol{\omega}_{\mathcal{A}}$.

1. $0 \leq \omega_A \cdot \omega_B \leq 1$ *(notion of orthogonality),*
2. $\omega_A \cdot \omega_B = \omega_B \cdot \omega_A$
3. $\omega_A \cdot \omega_B \leq 1 - \frac{1}{4} \|\omega_A - \omega_B\|^2$; *equality holds iff* $\omega$ *is pure* (usual sense)
4. $\omega_A, \omega_B \mapsto \omega_A \cdot \omega_B$ *is locally continuous*
5. *there are complete families of orthogonal states* $\{\omega_{A_m} \in \boldsymbol{\omega}_{\mathcal{A}}\}_{m \in \mathbb{N}}$*, i.e.* $\sum_m \omega_B \cdot \omega_{A_m} = 1$ *for any* $\omega_B \in \boldsymbol{\omega}_{\mathcal{A}}$.

# Primitive observables

**Question:** How can one relate these transition probabilities to observations?

Recall: $\omega_A \in \boldsymbol{\omega}_{\boldsymbol{\mathcal{A}}}$, non-mixing operations $B \in \boldsymbol{\mathcal{A}}$,

$$\omega_A \mapsto (1/\omega_A(B^* B))\, \omega_A \circ \mathrm{Ad}\, B \,.$$

Restrict operations $B$ to unitary operators $U$ (observable); result

$$\omega_A \mapsto \omega_A \circ \mathrm{Ad}\, U = \omega_{UA}\,, \quad \omega_A \in \boldsymbol{\omega}_{\boldsymbol{\mathcal{A}}}\,.$$

Examples: effects of temporary perturbation of dynamics

**Transition probability** (fidelity of operation):

$$\omega_A \cdot (\omega_A \circ \mathrm{Ad}\, U) = \omega_A \cdot \omega_{UA} = |\omega_A(U)|^2\,.$$

Can be observed by measurements of $U$ in state $\omega_A$.

## Definition

Primitive observables are fixed by unitaries $U \in \mathcal{A}$. For given $\omega_A \in \boldsymbol{\omega}_{\mathcal{A}}$

- $\omega_A \mapsto \omega_{UA}$ describes the effect of the corresponding operation
- $\omega_A \cdot \omega_{UA} = |\omega_A(U)|^2$ is the fidelity of this operation

Example: $U = E + t(1 - E)$ with $E$ projection, $t \in \mathbb{T}$. Fidelity

$$\omega_A \cdot \omega_{UA} = \omega_A(E)^2 + \omega_A(1-E)^2 + 2\,\mathrm{Re}(t)\,\omega_A(E)\omega_A(1-E)$$

Standard expectation values of observables can be recovered:

## Proposition

*Given projection $E \in \mathcal{A}$, (finite number of) states $\omega_A \in \boldsymbol{\omega}_{\mathcal{A}}$, and $\varepsilon > 0$. There exists a unitary $U \in \mathcal{A}$*

1. $|\omega_A \cdot \omega_{UA} - \omega_A(E)^2| < \varepsilon$, *i.e. "usual probatilities $\approx \sqrt{\text{fidelities}}$"*
2. $\omega_{UA}(1 - E) < \varepsilon$ *(compare von Neumann projection postulate)*

**Question:** Is $\omega_A \cdot \omega_B$ operationally defined for any $\omega_A, \omega_B \in \boldsymbol{\omega_{\mathcal{A}}}$?
(This requires that there are unitaries $U \in \mathcal{A}$ such that $\|\omega_B - \omega_{UA}\| < \varepsilon$.)

**Theorem (Connes, Haagerup, Størmer)**

*Let $\omega$ be of type $III_\lambda$ and let*

1. $0 \leq \lambda < 1$. *There are* $\omega_A, \omega_B \in \boldsymbol{\omega_{\mathcal{A}}}$ *s.t.* $\inf_U \|\omega_B - \omega_{UA}\| > \varepsilon(\lambda)$.
2. $\lambda = 1$. *Then* $\inf_U \|\omega_B - \omega_{UA}\| = 0$ *for any* $\omega_A, \omega_B \in \boldsymbol{\omega_{\mathcal{A}}}$.

Concept of transition probabilities (operationally) meaningful for

- pure states $\omega$ on $\mathcal{A}$
- generic states $\omega$ on $\mathcal{A}$ of type $\mathrm{III}_1$.

These are exactly the two cases of interest in quantum field theory!

# Conclusions

*Features of time:*

- arrow of time is a fundamental fact (can be encoded in theory)
- statements about the past require some theory (are ambiguous)
- conflicts with quantum physics (modification of concepts needed)

*New look at quantum physics:*

- fixed algebra replaced by funnel of algebras
- generic states and their excitations replace concept of pure states
- superpositions defined, based on bijective lifts to funnel
- transition probabilities can be defined
- primitive (unitary) observables determine transition probabilities
- meaningful framework for states in QFT (type $\mathrm{I}_\infty$ and $\mathrm{III}_1$)