Classical dynamics, arrow of time, and genesis of the Heisenberg commutation relations

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Mathematics of interacting QFT models
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Background

Perturbative AQFT led to a new constructive scheme for quantum physics

Ingredients:

- classical systems, orbits in configuration space, Lagrangeans
- operations (perturbations of system), labelled by functionals on orbits (fixed by potentials, durations in time)
- arrow (direction) of time; entering into the microworld by order (succession) of operations

Result: dynamical C*-algebra for given Lagrangean; commutation relations *etc* arise from its intrinsic structure.

New look at quantum physics; no *a priori* "quantization rules"

This talk: application of scheme to classical mechanics

Notation:

N particles in $\mathbb{R}^s$, equal masses, distinguishable

positions $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) \in \mathbb{R}^{sN}$

continuous orbits $\boldsymbol{x} : \mathbb{R} \to \mathbb{R}^{sN}$, family denoted by $\mathscr{C}$,

loops $\boldsymbol{x}_0 : \mathbb{R} \to \mathbb{R}^{sN}$ with compact support, form vector space $\mathscr{C}_0 \subset \mathscr{C}$,

velocities $\dot{\boldsymbol{x}} : \mathbb{R} \to \mathbb{R}^{sN}$

Perturbations:

(given by potentials, time dependencies) are described by space $\mathscr{F}$ of functionals $F : \mathscr{C} \to \mathbb{R}$

$$F[\boldsymbol{x}] \doteq \int dt\, F(t, \boldsymbol{x}(t))$$
$$t, \boldsymbol{x} \mapsto F(t, \boldsymbol{x}) = \boldsymbol{f}_0(t)\, \boldsymbol{x} + \sum_k g_k(t)\, V_k(\boldsymbol{x})$$

where $\boldsymbol{f}_0 \in \mathscr{C}_0$, $g_k \in \mathcal{D}(\mathbb{R}^s)$, V_k continuous, bounded

Support of F: union of supports of underlying test functions

Shifts: $F \mapsto F^{\boldsymbol{x}_0}$, $\boldsymbol{x}_0 \in \mathscr{C}_0$ given by $\boldsymbol{x} \mapsto F^{\boldsymbol{x}_0}[\boldsymbol{x}] \doteq F[\boldsymbol{x} + \boldsymbol{x}_0]$

Lagrangeans:

$$t \mapsto \mathcal{L}(\boldsymbol{x}(t)) \doteq (1/2)\,\dot{\boldsymbol{x}}(t)^2 - V(\boldsymbol{x}(t))\,, \quad \boldsymbol{x} \in \mathscr{C}\,;$$

action $\int dt \mathcal{L}(\boldsymbol{x}(t))$; relative action for loops $\boldsymbol{x}_0 \in \mathscr{C}_0$:

$$\delta\mathcal{L}(\boldsymbol{x}_0)[\boldsymbol{x}] = \int dt\, \chi(t)\big(\mathcal{L}(\boldsymbol{x}(t) + \boldsymbol{x}_0(t)) - \mathcal{L}(\boldsymbol{x}(t))\big)$$

with $\chi \restriction \mathrm{supp}\, \boldsymbol{x}_0 = 1$ (**note:** element of $\mathscr{F}$, linear term $\boldsymbol{x}$ appears)

Stationary points of action: Euler-Lagrange equation

$$\ddot{\boldsymbol{x}}(t) + \partial V(\boldsymbol{x}(t)) = 0$$

Propagators: "inverses" of $K = -\frac{d^2}{dt^2}$ *i.e.* $K\Delta_\bullet = \Delta_\bullet K = 1$

advanced Δ_A, retarded Δ_R, mean $\Delta_D \doteq (1/2)(\Delta_A + \Delta_R)$

commutator function: $\Delta \doteq \Delta_R - \Delta_A$, $K\Delta = \Delta K = 0$

Dynamical algebra

Step 1: given a Lagrangean $\mathcal{L}$, construct a dynamical group $\mathcal{G}_{\mathcal{L}}$

Definition: $\mathcal{G}_{\mathcal{L}}$ is the free group generated by symbols $S_{\mathcal{L}}(F)$, $F \in \mathscr{F}$, modulo the relations

(i) $S_{\mathcal{L}}(F) = S_{\mathcal{L}}(F^{\boldsymbol{x}_0} + \delta\mathcal{L}(\boldsymbol{x}_0))$ for all $F \in \mathscr{F}$, $\boldsymbol{x}_0 \in \mathscr{C}_0$

(ii) $S_{\mathcal{L}}(F_1)S_{\mathcal{L}}(F_2) = S_{\mathcal{L}}(F_1 + F_2)$ whenever F_1 has support in the future of F_2

Remarks: the elements of $\mathcal{G}_{\mathcal{L}}$ describe the effects of perturbations on the underlying system without stipulating their concrete action

(i) $F = 0$ implies $S_{\mathcal{L}}(\delta\mathcal{L}(\boldsymbol{x}_0)) = S(0) = 1$ (Euler-Lagrange equation)

(ii) constant functionals $F_{\mathrm{h}} : \mathscr{F} \to \mathrm{h} \in \mathbb{R}$ have arbitrary support in time; thus $S(F)S(F_{\mathrm{h}}) = S(F + F_{\mathrm{h}}) = S(F_{\mathrm{h}})S(F)$ (form central subgroup)

Step 2: proceed from $\mathcal{G}_{\mathcal{L}}$ to *-algebra $\mathcal{A}_{\mathcal{L}}$

sums: $\sum cS$, $c \in \mathbb{C}$, $S \in \mathcal{G}_{\mathcal{L}}$ span $\mathcal{A}_{\mathcal{L}}$

adjoints: $(\sum cS)^* \doteq \sum \overline{c} S^{-1}$ (S unitary operators)

products: fixed by distributive law

fixing scale: $S(F_{\mathrm{h}}) = e^{i\mathrm{h}} 1$, $h \in \mathbb{R}$ (amounts to atomic units)

norm: algebra has faithful states and thus a (maximal) C*-norm

Definition: Given $\mathscr{L}$, the corresponding dynamical algebra $\mathcal{A}_{\mathcal{L}}$ is the C*-algebra determined by the dynamical group $\mathcal{G}_{\mathcal{L}}$.

No quantization conditions, functional integrals *etc* ; only classical concepts used ("common language", cf. Bohr's doctrine)

Derivation of Heisenberg relations

Consider non-interacting Lagrangean

$$t \mapsto \mathcal{L}_0(\boldsymbol{x}(t)) = (1/2)\,\dot{\boldsymbol{x}}(t)^2$$

Simplest (linear) perturbations $\langle \boldsymbol{f}_0, \boldsymbol{x} \rangle \doteq \int dt\, \boldsymbol{f}_0(t)\, \boldsymbol{x}(t)$

$$\boldsymbol{x} \mapsto F_{\boldsymbol{f}_0}[\boldsymbol{x}] \doteq \langle \boldsymbol{f}_0, \boldsymbol{x} \rangle + (1/2)\langle \boldsymbol{f}_0, \Delta_D \boldsymbol{f}_0 \rangle\,, \quad \boldsymbol{f}_0 \in \mathscr{C}_0$$

Definition: $W(\boldsymbol{f}_0) \doteq S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0}),\ \ \boldsymbol{f}_0 \in \mathscr{C}_0$.

Theorem

(1) $W(K\boldsymbol{x}_0) = 1, \quad \boldsymbol{x}_0 \in \mathscr{C}_0$

(2) $W(\boldsymbol{f}_0)W(\boldsymbol{g}_0) = e^{-(i/2)\langle \boldsymbol{f}_0, \Delta \boldsymbol{g}_0 \rangle} W(\boldsymbol{f}_0 + \boldsymbol{g}_0), \quad \boldsymbol{f}_0, \boldsymbol{g}_0 \in \mathscr{C}_0$

Interpretation: Weyl operators $W(\boldsymbol{x}_0) \doteq e^{i\langle \boldsymbol{x}_0, \boldsymbol{Q} \rangle}$, $\boldsymbol{x}_0 \in \mathscr{C}_0$

(1) generators solutions of Heisenberg eq.: $t \mapsto \boldsymbol{Q}(t) = \boldsymbol{Q} + t\dot{\boldsymbol{Q}}$

(2) $[\boldsymbol{Q}_k, \dot{\boldsymbol{Q}}_l] = i\delta_{kl}1, \quad [\boldsymbol{Q}_k, \boldsymbol{Q}_l] = [\dot{\boldsymbol{Q}}_k, \dot{\boldsymbol{Q}}_l] = 0$

Physics: operators of position $\boldsymbol{Q}$ and momentum $\boldsymbol{P} \doteq \dot{\boldsymbol{Q}}$

Derivation of Heisenberg relations

Proofs:

(1) (dynamics) $\boldsymbol{x}_0 \in \mathscr{C}_0$ recall: $K = -\frac{d^2}{dt^2}$

$$\begin{aligned}\boldsymbol{x} \mapsto F_{K\boldsymbol{x}_0}[\boldsymbol{x}] &= \langle K\boldsymbol{x}_0, \boldsymbol{x}\rangle + (1/2)\langle K\boldsymbol{x}_0, \Delta_D K\boldsymbol{x}_0\rangle \\ &= \langle \dot{\boldsymbol{x}}_0, \dot{\boldsymbol{x}}\rangle + (1/2)\langle \dot{\boldsymbol{x}}_0, \dot{\boldsymbol{x}}_0\rangle = \delta\mathcal{L}_0(\boldsymbol{x}_0)[\boldsymbol{x}]\end{aligned}$$

hence $S_{\mathcal{L}_0}(F_{K\boldsymbol{x}_0}) = S_{\mathcal{L}_0}(\delta\mathcal{L}_0(\boldsymbol{x}_0)) = S_{\mathcal{L}_0}(0) = 1$;
similarly $S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+K\boldsymbol{x}_0}) = S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0})$

(2) (Weyl relations) Given $\boldsymbol{f}_0, \boldsymbol{g}_0$, let $\boldsymbol{f}_0 + K\boldsymbol{x}_0$ be later than $\boldsymbol{g}_0$. Then
$S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0})S_{\mathcal{L}_0}(F_{\boldsymbol{g}_0}) = S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+K\boldsymbol{x}_0})S_{\mathcal{L}_0}(F_{\boldsymbol{g}_0}) = S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+K\boldsymbol{x}_0} + F_{\boldsymbol{g}_0})$.

Linearity of $F_{\boldsymbol{f}}[\boldsymbol{x}]$ with regard to $\boldsymbol{x}$ implies

$$\begin{aligned}S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+K\boldsymbol{x}_0} + F_{\boldsymbol{g}_0}) &= S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+K\boldsymbol{x}_0+\boldsymbol{g}_0} + F_{\mathrm{h}_{\boldsymbol{f}_0+K\boldsymbol{x}_0,\boldsymbol{g}_0}}) \\ &= e^{i\mathrm{h}_{\boldsymbol{f}_0+K\boldsymbol{x}_0,\boldsymbol{g}_0}} S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+K\boldsymbol{x}_0+\boldsymbol{g}_0}) = e^{i\mathrm{h}_{\boldsymbol{f}_0+K\boldsymbol{x}_0,\boldsymbol{g}_0}} S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+\boldsymbol{g}_0})\end{aligned}$$

where

$$\mathrm{h}_{\boldsymbol{f}_0+K\boldsymbol{x}_0,\boldsymbol{g}_0} = \cdots = -(1/2)\langle \boldsymbol{f}_0 + K\boldsymbol{x}_0, \Delta\boldsymbol{g}_0\rangle = -(1/2)\langle \boldsymbol{f}_0, \Delta\boldsymbol{g}_0\rangle\,. \qquad \checkmark$$

Proofs:

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$$\boldsymbol{x} \mapsto F_{K\boldsymbol{x}_0}[\boldsymbol{x}] = \langle K\boldsymbol{x}_0, \boldsymbol{x}\rangle + (1/2)\langle K\boldsymbol{x}_0, \Delta_D K\boldsymbol{x}_0\rangle$$
$$= \langle \dot{\boldsymbol{x}}_0, \dot{\boldsymbol{x}}\rangle + (1/2)\langle \dot{\boldsymbol{x}}_0, \dot{\boldsymbol{x}}_0\rangle = \delta\mathcal{L}_0(\boldsymbol{x}_0)[\boldsymbol{x}]$$

hence $S_{\mathcal{L}_0}(F_{K\boldsymbol{x}_0}) = S_{\mathcal{L}_0}(\delta\mathcal{L}_0(\boldsymbol{x}_0)) = S_{\mathcal{L}_0}(0) = 1$;
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$$= e^{i\mathrm{h}_{\boldsymbol{f}_0+K\boldsymbol{x}_0,\boldsymbol{g}_0}}\, S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+K\boldsymbol{x}_0+\boldsymbol{g}_0}) = e^{i\mathrm{h}_{\boldsymbol{f}_0+K\boldsymbol{x}_0,\boldsymbol{g}_0}}\, S_{\mathcal{L}_0}(F_{\boldsymbol{f}_0+\boldsymbol{g}_0})$$

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Change of Lagrangean (potentials V as before)

$$t \mapsto \mathcal{L}(\boldsymbol{x}(t)) = \mathcal{L}_0(\boldsymbol{x}(t)) - V(\boldsymbol{x}(t))$$

temporary perturbation (χ smooth characteristic function)

$$t \mapsto \mathcal{L}_\chi(\boldsymbol{x}(t)) \doteq \mathcal{L}_0(\boldsymbol{x}(t)) - \underbrace{\chi(t) V(\boldsymbol{x}(t))}_{V_\chi(t,\boldsymbol{x}(t))}$$

Definition: (cf. relative scattering matrices)

$$S_{\mathcal{L}_\chi}(F) \doteq S_{\mathcal{L}_0}(-V_\chi)^{-1} S_{\mathcal{L}_0}(F - V_\chi) \in \mathcal{A}_{\mathcal{L}_0}, \quad F \in \mathscr{F}$$

Properties: (elementary computation)

$$S_{\mathcal{L}_\chi}(F^{\boldsymbol{x}_0} + \delta\mathcal{L}_\chi(\boldsymbol{x}_0)) = S_{\mathcal{L}_\chi}(F) \qquad \textbf{(i)}$$

$$S_{\mathcal{L}_\chi}(F_1) S_{\mathcal{L}_\chi}(F_2) = S_{\mathcal{L}_\chi}(F_1 + F_2) \quad \text{if } F_1 \text{ is later than } F_2 \qquad \textbf{(ii)}$$

Conclusion: defining relations for dynamical algebra $\mathcal{A}_{\mathcal{L}_\chi} \simeq \mathcal{A}_{\mathcal{L}_0}$

Goal: limit $\chi \to 1$ (global dynamics)
Note: Let $\mathbb{I} \subset \mathbb{R}$ and $\chi \upharpoonright \mathbb{I} = 1$, then $\delta\mathcal{L}_\chi(\boldsymbol{x}_0) = \delta\mathcal{L}(\boldsymbol{x}_0)$ if $\boldsymbol{x}_0 \in \mathscr{C}_0(\mathbb{I})$

Definition: $\mathcal{A}_{\mathcal{L}_\chi}(\mathbb{I})$ algebra generated by $S_{\mathcal{L}_\chi}(F)$, $F \in \mathscr{F}(\mathbb{I})$.

Observation: $\mathcal{A}_{\mathcal{L}_\chi}(\mathbb{I}) \simeq \mathcal{A}_{\mathcal{L}}(\mathbb{I})$ and algebras $\mathcal{A}_{\mathcal{L}_\chi}(\mathbb{I})$ for different χ are related by inner automorphisms of $\mathcal{A}_{\mathcal{L}_0}$

Detailed analysis: for increasing intervals $\mathbb{I}_n$ and functions χ_n there exist injective homomorphisms $\beta_n : \mathcal{A}_{\mathcal{L}}(\mathbb{I}_n) \to \mathcal{A}_{\mathcal{L}_0}(\mathbb{I}_{n+1})$ such that

$$\gamma \doteq \lim_n \beta_n$$

point-wise in norm on $\mathcal{A}_{\mathcal{L}} = \overline{\bigcup_{\mathbb{I}} \mathcal{A}_{\mathcal{L}}(\mathbb{I})}$.

Theorem

Let $\mathcal{L}_0, \mathcal{L}$ be Lagrangeans. There exist monomorphisms $\gamma : \mathcal{A}_{\mathcal{L}} \to \mathcal{A}_{\mathcal{L}_0}$ such that $\gamma(\mathcal{A}_{\mathcal{L}}(\mathbb{I})) \subset \mathcal{A}_{\mathcal{L}_0}(\widehat{\mathbb{I}})$ for any $\mathbb{I}$ and bounded $\widehat{\mathbb{I}} \supset \mathbb{I}$.

Consider Schrödinger representation of $\boldsymbol{Q}, \boldsymbol{P}$ on $\mathcal{H}_S$ with dynamics $\mathcal{L}_0$
Claim: operators $S(F)$ are represented by time ordered exponentials.

Problem: For $F \in \mathscr{F}$, determine $T(F) \doteq T\, e^{i \int_{-\infty}^{\infty} dt\, F(\boldsymbol{Q}+t\boldsymbol{P})}$

- bounded functionals F_b: Dyson expansion

$$T(F_b) = 1 + \sum_k i^k \int_{-\infty}^{\infty} dt_1 \,\ldots \int_{-\infty}^{t_{k-1}} dt_k\, F_b(\boldsymbol{Q} + t_1 \boldsymbol{P}_1) \cdots F_b(\boldsymbol{Q} + t_k \boldsymbol{P}_1)$$

- linear (unbounded) L: solution of linear differential equation

$$T(L_{\boldsymbol{f}_0}) = e^{i \int dt\, \boldsymbol{f}_0(t)(\boldsymbol{Q}+t\boldsymbol{P})}\, e^{-(i/2)\langle \boldsymbol{f}_0, \Delta_D \boldsymbol{f}_9\rangle} = W(\boldsymbol{f}_0)\, e^{-(i/2)\langle \boldsymbol{f}_0, \Delta_D \boldsymbol{f}_9\rangle}$$

- combination: $\overline{T}(F_b + L_{\boldsymbol{f}_0}) \doteq T(F_b^{-\Delta_A \boldsymbol{f}_0})\, T(L_{\boldsymbol{f}_0})$

Ansatz based on results of structural analysis; it has all required properties

Proof: E.g. "dynamical relation" for bounded functionals F_b

$$\overline{T}(F_b^{\boldsymbol{x}_0} + \delta\mathcal{L}_0(\boldsymbol{x}_0)) = \overline{T}(F_b^{\boldsymbol{x}_0} + F_{K\boldsymbol{x}_0}) = \overline{T}(\overbrace{F_b^{\boldsymbol{x}_0} + F_\mathrm{h}} + L_{K\boldsymbol{x}_0})$$
$$= T(F_b^{\boldsymbol{x}_0 - \Delta_A K \boldsymbol{x}_0})\, e^{i\mathrm{h}}\, T(L_{K\boldsymbol{x}_0}) = T(\underbrace{F_b^{\boldsymbol{x}_0 - \Delta_A K \boldsymbol{x}_0}}_{F_b})\, \underbrace{T(F_{K\boldsymbol{x}_0})}_{1} = \overline{T}(F_b) \quad \checkmark$$

Definition: Representation $(\pi_S, \mathcal{H}_S)$ of $\mathcal{A}_{\mathcal{L}_0}$ fixed by putting

$$\pi_S(S_{\mathcal{L}_0}(F)) \doteq \overline{T}(F), \qquad F \in \mathscr{F}.$$

Other algebras $\mathcal{A}_\mathcal{L}$ are represented by $(\pi, \mathcal{H}_S)$, where $\pi \doteq \pi_s \circ \gamma$

Theorem

(i) The representations $(\pi, \mathcal{H}_S)$ of $\mathcal{A}_\mathcal{L}$ are "regular" and irreducible

(ii) This holds also true for $\pi \upharpoonright \mathcal{A}_\mathcal{L}(\mathbb{I})$ for any finite interval $\mathbb{I}$

Task: Using only operations, (i) compute probability that a state has the property described by a projection E and (ii) determine properties of the state after the operation.

Definition: Let $(\pi, \mathcal{H})$ be irreducible representation of $\mathcal{A}_{\mathcal{L}}$, let $\Omega \in \mathcal{H}$, and consider vector state $\omega(\,\cdot\,) = \langle \Omega, \pi(\,\cdot\,)\,\Omega \rangle$ on $\mathcal{A}_{\mathcal{L}}$. The operations $S \in \mathcal{A}_{\mathcal{L}}$ induce maps $\omega \mapsto \omega_S \doteq \omega \circ \mathrm{Ad}\, S^{-1}$

Transition probability: ("fidelity of operation")

$$\omega \cdot \omega_S \doteq |\langle \Omega, \pi(S)\Omega \rangle|^2 = |\omega(S)|^2$$

Theorem

Let $\mathcal{H}_N \subset \mathcal{H}$ be finite dimensional, let E be infinite projection, and let $\varepsilon > 0$. There exists unitary operator $S_\varepsilon \in \mathcal{A}_{\mathcal{L}}$ such that for any $\Omega \in \mathcal{H}_N$

$$|\omega \cdot \omega_{S_\varepsilon} - \omega(E)^2| < \varepsilon\,, \quad \omega_{S_\varepsilon}(1 - E) < \varepsilon\,.$$

Note: **no** collapse of wave functions (Lüders, von Neumann); suitable operations determine "primitive observables" [DB, E. Størmer]

Conclusions

New look at quantum mechanics, based on classical concepts

- system: configuration space, orbits, Lagrangean
- operations: perturbations of system (with or without observer)
- time: directed; its arrow matters already in microphysics

Effect of operations on system

- described by dynamical group (composition of operations)
- extension to C*-algebra standard procedure
- no quantization rules; non-commutativity due to arrow of time

Consequences

- commutation relations, familiar framework recovered
- representation theory based on time ordered products
- statistical interpretation can be deduced from operations

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