Measurement schemes for quantum field theory in curved spacetimes

CJ Fewster
University of York

Mathematics of interacting QFT models
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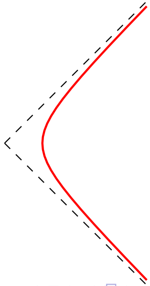
arXiv:1810.06512 - Joint work with Rainer Verch; short summary in arXiv:1904.06944

Motivation: Waiting for Unruh CJF, Juárez-Aubry & Louko, 2016

Unruh effect (1976)

Thermalisation of an eternally uniformly accelerated detector in Minkowski spacetime, coupled to a QFT in the vacuum state.

- Robust prediction of QFT – holds for many states, rigorous nonperturbative proof de Bièvre and Merkli (2006)
- Closely related to the Hawking effect



Detailed balance relation

$$p(+) = e^{-\beta \Delta E} p(-)$$

for a 2-level detector with energy gap ΔE.

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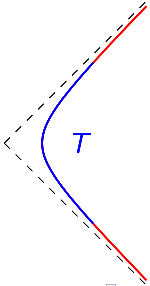
How long does thermalisation take?

- Couple detector for time T
- Approx. detailed balance requires T to grow faster than polynomially in ΔE
- You have to wait for Unruh!



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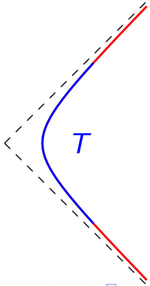
Wald's question

Where do you wait for Unruh?

- Is detailed balance seen at finite times, or only at infinity?



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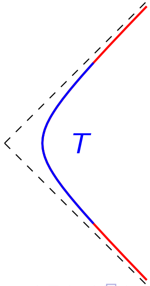
Wald's question is natural.

But the UdW literature does not discuss probe measurement, nor its interpretation as a measurement of the field.



In fact the problem is wider.

A gap, and our goals

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- How is a measurement of a probe expressed in terms of a local QFT observable?
- How does the state change after selective measurement?
- What (if anything) is the analogue of 'instantaneous collapse'?
- Can all this be adapted covariantly to curved spacetimes?

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Aim: Develop QMT for AQFT, to provide better operational foundations for both subjects, and giving concrete results in models.

Take measurement theory out of Hilbert space,
and put it back in spacetime, where it belongs.

Measurement basics

One part of the world (probe) is used to learn about another (system).

This relies on there being a coupling between probe and system which can be controlled to some extent.

We should be able, at least to a good approximation, to

- prepare probe and system independently
- measure the probe in isolation from the system

Measurements are performed on the coupled system–probe set-up, but are described in the language of a fictitious uncoupled system.

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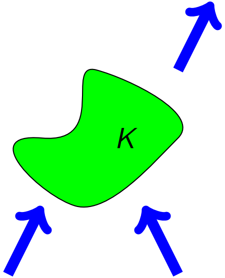
NB: We restrict to a single step in the measurement chain.

Quis metietur ipsos mensores? (HCE Fewster, after Juvenal)

Scenario

A QFT (system) is coupled to another QFT (probe) in a compact spacetime region K (a proxy for the experimental design). The probe is measured elsewhere.

Measure probe



Prepare system and probe

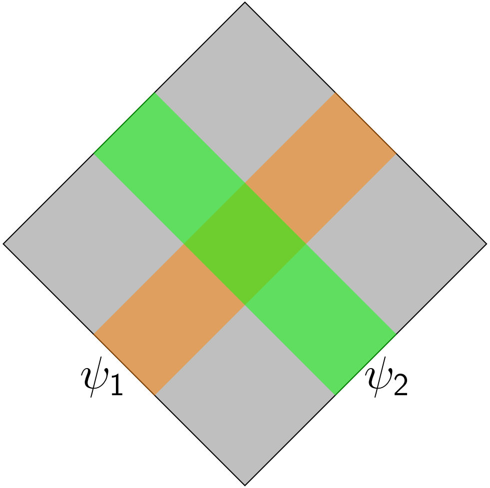
New state

$\overset{?}{\approx}$

Hypothetical local system measurement

Prepare system

Local modification of couplings in QFT



An interaction term

$$\psi_1\psi_2\varphi_1\varphi_2$$

provides a tunable coupling between φ_1 and φ_2.

Algebraic QFT

Describe a QFT on $\boldsymbol{M}$ in terms of a $*$-algebra $\mathcal{A}(\boldsymbol{M})$ with unit and subalgebras $\mathcal{A}(\boldsymbol{M}; N)$ for suitable open regions $N \subset \boldsymbol{M}$.

Minimal conditions

- **Isotony** $N_1 \subset N_2 \implies \mathcal{A}(\boldsymbol{M}; N_1) \subset \mathcal{A}(\boldsymbol{M}; N_2)$
- **Timeslice** $\mathcal{A}(\boldsymbol{M}; N) = \mathcal{A}(\boldsymbol{M})$ if N contains a Cauchy surface of $\boldsymbol{M}$
- **Einstein** $[\mathcal{A}(\boldsymbol{M}; N_1), \mathcal{A}(\boldsymbol{M}; N_2)] = 0$ if $N_{1,2}$ are causally disjoint

NB A given observable may be localisable in many distinct regions.

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A state is a positive, normalised linear functional $\omega : \mathcal{A}(\boldsymbol{M}) \to \mathbb{C}$, assigning an expectation value $\omega(A)$ to $A \in \mathcal{A}(\boldsymbol{M})$.

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NB A given observable may be localisable in many distinct regions.

$A \in \mathcal{A}(\boldsymbol{M}; N)$ is interpreted by fiat as

- an observable localisable in N if $A = A^*$
- an operation performable in N in general.

No discussion of how observables are measured or operations performed.

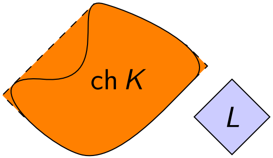
Coupled combinations and scattering

Describe both the system and the probe by AQFTs $\mathcal{A}$, $\mathcal{B}$ on $\boldsymbol{M}$.

Their uncoupled combination is described by $\mathcal{U} = \mathcal{A} \otimes \mathcal{B}$.

Theory $\mathcal{C}$ is a coupled combination of $\mathcal{A}$ and $\mathcal{B}$ with compact coupling region K.

$$\mathrm{ch}\,(K) = J^+(K) \cap J^-(K)$$



(Minimal) abstract definition:
$\forall L$ outside the causal hull $\mathrm{ch}\,(K)$
$\exists$ an isomorphism

$$\mathcal{U}(\boldsymbol{M}; L) \to \mathcal{C}(\boldsymbol{M}; L)$$

compatible with isotony.

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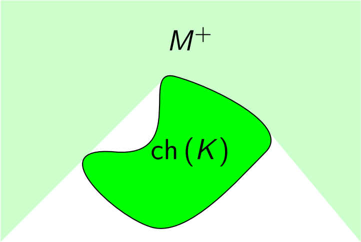

Defining in/out regions

$$M^\pm = M \setminus J^\mp(K)$$

we obtain isomorphisms

$$\tau^\pm : \mathcal{U}(\boldsymbol{M}) \to \mathcal{C}(\boldsymbol{M})$$

$$\tau^\pm : \mathcal{U}(\boldsymbol{M}) = \mathcal{U}(\boldsymbol{M}; M^\pm) \longrightarrow \mathcal{C}(\boldsymbol{M}; M^\pm) = \mathcal{C}(\boldsymbol{M})$$

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Upshot: covariantly described advanced/retarded response maps

$$\tau^{-/+} : \mathcal{U}(\boldsymbol{M}) \longrightarrow \mathcal{C}(\boldsymbol{M})$$

are isomorphisms identifying the uncoupled and coupled combinations at early/late times. The scattering map is

$$\Theta = (\tau^-)^{-1} \circ \tau^+ \in \mathrm{Aut}(\mathcal{U}(\boldsymbol{M}))$$

Locality: $\Theta \restriction \mathcal{U}(\boldsymbol{M}; N) = \mathrm{id}$, if $N \subset K^\perp$.

Measurement scheme: prepare early, measure late

Describe measurements of $\mathcal{C}(\boldsymbol{M})$ in uncoupled language.

Fixing a probe preparation state σ and system state ω, the state

$$\underset{\sim}{\omega}_\sigma = ((\tau^-)^{-1})^*(\omega \otimes \sigma)$$

of $\mathcal{C}(\boldsymbol{M})$ is uncorrelated at early times.

An observable $\widetilde{B} := \tau^+(\mathbf{1} \otimes B)$ tests probe d.o.f. at late times.

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Measurement of $\widetilde{B}$ in state $\underset{\sim}{\omega}_\sigma$ gives expectation value

$$\underset{\sim}{\omega}_\sigma(\widetilde{B}) = (\omega \otimes \sigma)(\Theta(\mathbf{1} \otimes B)) = \omega(\eta_\sigma(\Theta(\mathbf{1} \otimes B)))$$

where $\eta_\sigma : \mathcal{A}(\boldsymbol{M}) \otimes \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ linearly extends $A \otimes B \mapsto \sigma(B)A$.

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Definition: $\varepsilon_\sigma(B) = \eta_\sigma(\Theta(\mathbf{1} \otimes B))$ is the induced system observable corresponding to probe observable B.

Induced system observables

True and hypothetical expectation values agree, by construction

$$\underset{\sim}{\omega}_\sigma(\widetilde{B}) = \omega(\varepsilon_\sigma(B)) \qquad \text{for all } B \in \mathcal{B}(\boldsymbol{M}).$$

- In QMT language, $(\mathcal{C}, \tau^{\pm}, \sigma)$ is a measurement scheme for the system observables $\varepsilon_\sigma(B) \in \mathcal{A}(\boldsymbol{M})$ $(B \in \mathcal{B}(\boldsymbol{M}))$
- $\varepsilon_\sigma : \mathcal{B}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{M})$ is linear, completely positive, and obeys

$$\varepsilon_\sigma(\mathbf{1}) = \mathbf{1}, \qquad \varepsilon_\sigma(B^*) = \varepsilon_\sigma(B)^*, \qquad \varepsilon_\sigma(B)^*\varepsilon_\sigma(B) \leq \varepsilon_\sigma(B^*B).$$

Consequently, the true measurement displays greater variance than the hypothetical one due to detector fluctuations

$$\mathrm{Var}(\varepsilon_\sigma(B); \omega) \leq \mathrm{Var}(\widetilde{B}; \underset{\sim}{\omega}_\sigma).$$

Localisation

Recall: Θ acts trivially on $\mathcal{U}(\boldsymbol{M}; L)$ if $L \subset K^\perp$.

Theorem (a) If $B \in \mathcal{B}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then

$$\varepsilon_\sigma(B) = \eta_\sigma(\Theta(\mathbf{1} \otimes B)) = \eta_\sigma(\mathbf{1} \otimes B) = \sigma(B)\mathbf{1}.$$

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(b) If $A \in \mathcal{A}(\boldsymbol{M}; L)$ with $L \subset K^\perp$ then, for any B,

$$[\varepsilon_\sigma(B), A] = [\eta_\sigma(\Theta(\mathbf{1} \otimes B)), A] = \eta_\sigma(\Theta[\mathbf{1} \otimes B, A \otimes \mathbf{1}]) = 0$$

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Corollary If $\mathcal{A}$ obeys a Haag property,

$$\varepsilon_\sigma(B) \in \mathcal{A}(\boldsymbol{M}; L) \qquad \text{for all } B \in \mathcal{B}(\boldsymbol{M}),$$

where L is any open connected causally convex set containing K.

Effects and effect-valued measures

An effect is an observable s.t. B and $\mathbf{1} - B$ are positive, corresponding to a true/false measurement

$$\mathrm{Prob}(B \mid \omega) = \omega(B), \qquad \mathrm{Prob}(\neg B \mid \omega) = \omega(\mathbf{1} - B)$$

QMT refines observables to effect-valued measures (EVMs)

$$\mathsf{E} : \mathcal{X} \to \mathrm{Effects}(\mathcal{B}(\boldsymbol{M}))$$

for σ-algebra $\mathcal{X}$, with the interpretation

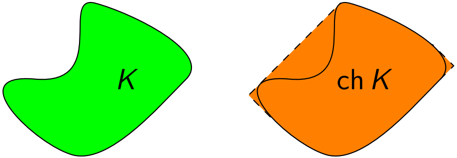
$$\omega(\mathsf{E}(X)) = \text{probability a result in range } X \text{ is observed in state } \omega$$

Each probe EVM induces a system EVM $\varepsilon_\sigma \circ \mathsf{E}$ (generally unsharp).

Summary so far

To every local probe observable B and probe preparation state σ there is an induced system observable, $\varepsilon_\sigma(B)$ which can be localised in any connected region containing the causal hull of K

$$\mathrm{ch}\,(K) = J^+(K) \cap J^-(K)$$



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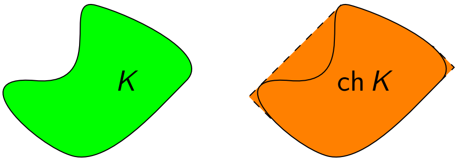

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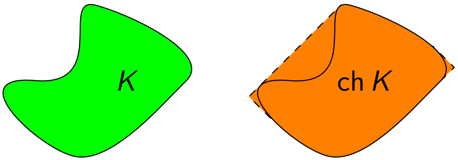

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Probe observables localisable in $K^\perp$ induce trivial system observables.

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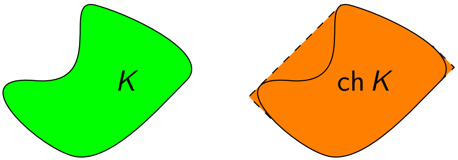
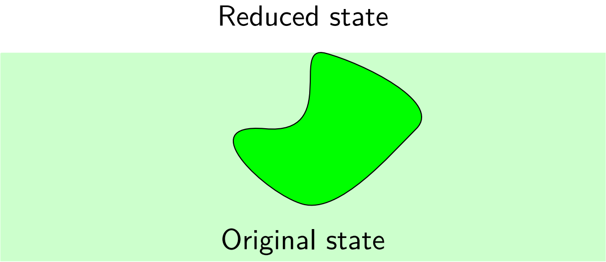


Expectation values of the true and hypothetical measurements match, but the true measurement has greater variance.

Justifies regarding $A \in \mathcal{A}(\boldsymbol{M}; L)$ as 'measureable within' L in the sense that the system and probe are coupled there.

States and instruments

Idealised instantaneous state reduction is potentially frame dependent if 'instantaneous' means 'constant time'.



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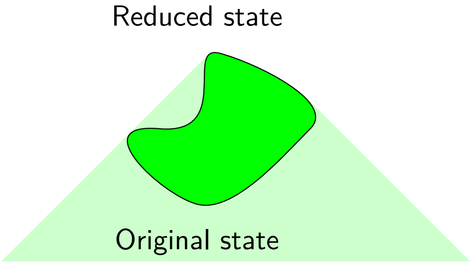

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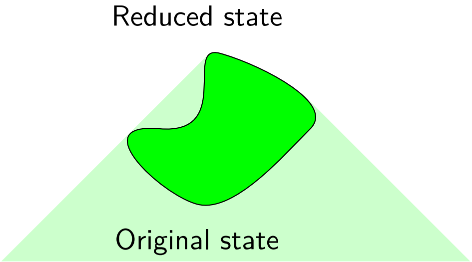

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Our framework permits the post-selected state to be calculated.

Post-selection and pre-instruments

Suppose a probe-effect B is tested when the system state is ω.

The post-selected system state, conditioned on the effect being observed, should correctly predict the probability of any system effect being observed, given that B was.

Post-selection and pre-instruments

Probability of a joint successful measurement of system effect A and probe effect B is

$$\mathrm{Prob}(A\&B) = \omega(\eta_\sigma(\Theta(A \otimes B)))$$

so
$$\mathrm{Prob}(A|B) = \frac{\mathrm{Prob}(A\&B)}{\mathrm{Prob}(B)} = \frac{(\mathcal{I}_\sigma(B)(\omega))(A)}{(\mathcal{I}_\sigma(B)(\omega))(\mathbf{1})},$$

where
$$(\mathcal{I}_\sigma(B)(\omega))(A) := (\omega \otimes \sigma)(\Theta(A \otimes B)).$$

Call $\mathcal{I}_\sigma(B) : \mathcal{A}(\boldsymbol{M})^*_+ \to \mathcal{A}(\boldsymbol{M})^*_+$ a pre-instrument.

If defined, the normalized post-selected state, conditioned on B, is

$$\omega' = \frac{\mathcal{I}_\sigma(B)(\omega)}{(\mathcal{I}_\sigma(B)(\omega))(\mathbf{1})}.$$

Locality and post-selection

For A localisable in $K^\perp$, the pre-instrument may be rewritten

$$\mathcal{J}_\sigma(B)(\omega)(A) = \omega(\eta_\sigma(\Theta(A \otimes B))) = \omega(A\varepsilon_\sigma(B))$$

so also
$$\omega'(A) = \frac{\omega(A\varepsilon_\sigma(B))}{\omega(\varepsilon_\sigma(B))}.$$

Theorem $\omega'(A) = \omega(A)$ iff A is uncorrelated with $\varepsilon_\sigma(B)$ in ω.

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Equality or otherwise of expectation values is not determined by the localisation region of A. E.g., if ω has a Reeh–Schlieder property, and A can be localised in $K^\perp$ then

$$\omega'(A) = \omega(A) \implies \varepsilon_\sigma(B) = \omega(\varepsilon_\sigma(B))\mathbf{1}$$

Post-selection on any nontrivial measurement alters expectation values in $K^\perp$ [and the rest of $\boldsymbol{M}$ – including the past]. This is attributable to correlation.

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No need to declare that ω changes to ω' across a surface in $\boldsymbol{M}$.

Successive measurement of two probes

For $i = 1, 2$ consider $\mathcal{B}_i$ with coupling regions K_i and scattering morphisms Θ_i.

Also consider $\mathcal{B}_1 \otimes \mathcal{B}_2$ as a combined probe with coupling region $K_1 \cup K_2$ and morphism $\hat{\Theta}$.

Suppose $K_2 \cap J^-(K_1) = \emptyset$, so K_2 is later than K_1 according to some observers and assume that causal factorisation holds, i.e.,

$$\hat{\Theta} = \hat{\Theta}_1 \circ \hat{\Theta}_2, \quad \text{where } \hat{\Theta}_1 = \Theta_1 \otimes_3 \mathrm{id} \quad \text{and } \hat{\Theta}_2 = \Theta_2 \otimes_2 \mathrm{id}$$

Theorem Coherence of successive measurement

$$\mathcal{I}_{\sigma_2}(B_2) \circ \mathcal{I}_{\sigma_1}(B_1) = \mathcal{I}_{\sigma_1 \otimes \sigma_2}(B_1 \otimes B_2)$$

Post-selection on B_1 and then B_2 agrees with post-selection on $B_1 \otimes B_2$.

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Also consider $\mathcal{B}_1 \otimes \mathcal{B}_2$ as a combined probe with coupling region $K_1 \cup K_2$ and morphism $\hat{\Theta}$.

Suppose $K_2 \cap J^-(K_1) = \emptyset$, so K_2 is later than K_1 according to some observers and assume that causal factorisation holds, i.e.,

$$\hat{\Theta} = \hat{\Theta}_1 \circ \hat{\Theta}_2, \quad \text{where } \hat{\Theta}_1 = \Theta_1 \otimes_3 \mathrm{id} \quad \text{and } \hat{\Theta}_2 = \Theta_2 \otimes_2 \mathrm{id}$$

Corollary If K_1 and K_2 are causally disjoint,

$$\mathcal{J}_{\sigma_2}(B_2) \circ \mathcal{J}_{\sigma_1}(B_1) = \mathcal{J}_{\sigma_1 \otimes \sigma_2}(B_1 \otimes B_2) = \mathcal{J}_{\sigma_1}(B_1) \circ \mathcal{J}_{\sigma_2}(B_2)$$

Summary so far

General questions have been answered:

- induced local observables localised near coupling region
- derivation of post-selected states
- no need to posit state change across surfaces
- successive measurements are coherent

Now turn to a specific model in which induced observables can be computed.

Probe model

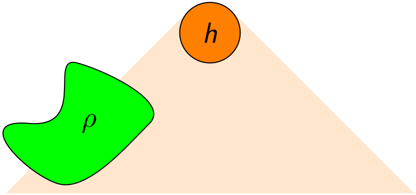
Two free scalar fields: Φ (system) and Ψ (probe) coupled via an interaction term

$$\mathcal{L}_{\text{int}} = -\rho\Phi\Psi, \qquad \rho \in C_0^\infty(M), \qquad K = \operatorname{supp}\rho.$$

Linear equations: standard quantisation applies at least for sufficiently weak coupling. As formal power series in $h \in C_0^\infty(M^+)$,

$$\Theta(\mathbf{1} \otimes e^{i\Psi(h)}) = e^{i\Phi(f^-)} \otimes e^{i\Psi(h^-)}$$

where f^- and $h^- - h$ are supported in $\operatorname{supp}\rho \cap J^-(\operatorname{supp} h)$.



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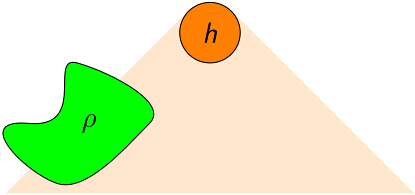


where f^- and $h^- - h$ are supported in $\operatorname{supp}\rho \cap J^-(\operatorname{supp} h)$.

$$\varepsilon_\sigma(e^{i\Psi(h)}) = \sigma\left(e^{i\Psi(h^-)}\right) e^{i\Phi(f^-)} = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

if σ is quasifree with two-point function S.

Examples of induced observables

$$\varepsilon_\sigma(e^{i\Psi(h)}) = e^{-S(h^-,h^-)/2} e^{i\Phi(f^-)}$$

$$\varepsilon_\sigma(\Psi(h)) = \Phi(f^-)$$
$$\varepsilon_\sigma(\Psi(h)^2) = \Phi(f^-)^2 + S(h^-, h^-)\mathbf{1}$$

Consequently,

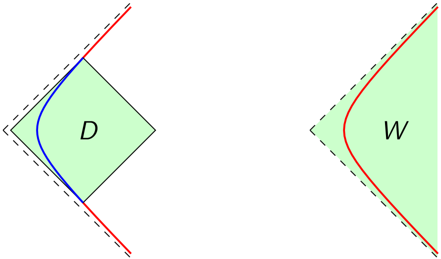
$$\mathbb{E}(\widetilde{\Psi(h)}; \underset{\sim}{\omega}_\sigma) = \omega(\Phi(f^-))$$
$$\mathrm{Var}(\widetilde{\Psi(h)}; \underset{\sim}{\omega}_\sigma) = \mathrm{Var}(\Phi(f^-); \omega) + S(h^-, h^-)$$

Increased variance in true measurement from detector fluctuations.

Localisation of induced observables

$\varepsilon_\sigma(\Psi(h)^n)$ may be localised in any open causally convex nhd of

$$\mathrm{supp}\, f^- \subset \mathrm{supp}\, \rho \cap J^-(\mathrm{supp}\, h)$$



Localisation region for finite-time coupling is a diamond D.

Localisation region for eternal coupling is a wedge W (can't do better).

Summary

- Operational framework of QMT adapted to AQFT
- Probe observables induce local system observables
- Localisation in the causal hull of coupling region
- Post-selected states, coherence under successive measurements
- No need to invoke state change across a surface
- Computation of induced observables for specific model

Lastly...

Summary

- Operational framework of QMT adapted to AQFT
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Lastly...

Thanks to Kasia and the admin team for the excellent organization of this meeting.