# The Kay-Wald theorem and HHI-like states on black hole space-times

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# Outline

# Unruh and Hawking effects

Two fundamental results in QFT in curved space-time

## Unruh effect

A uniformly accelerating observer in Minkowski space-time observes thermal radiation in the Minkowski vacuum

[ Fulling *PRD* **7** 2850 (1973); Davies *JPA* **8** 609 (1975); Unruh *PRD* **14** 870 (1976) ]

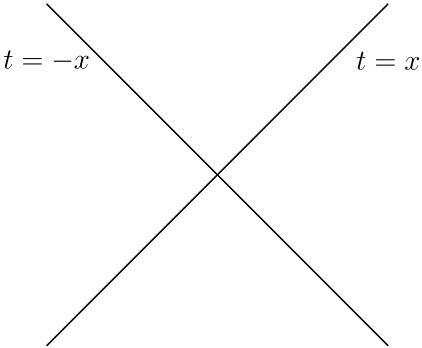
## Hawking effect

A black hole formed by gravitational collapse emits thermal radiation

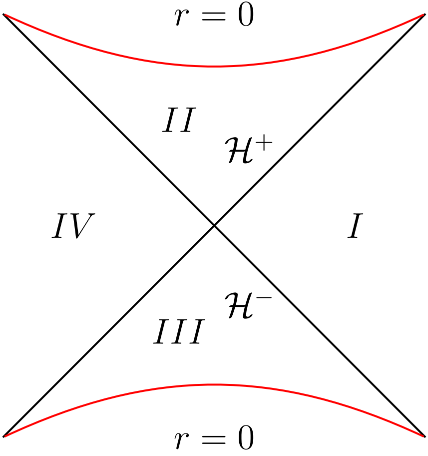
[ Hawking *CMP* **43** 199 (1975) ]

# An analogy [ Kay *CMP* **100** 57 (1985) ]

**Minkowski space-time**




**Kruskal space-time**



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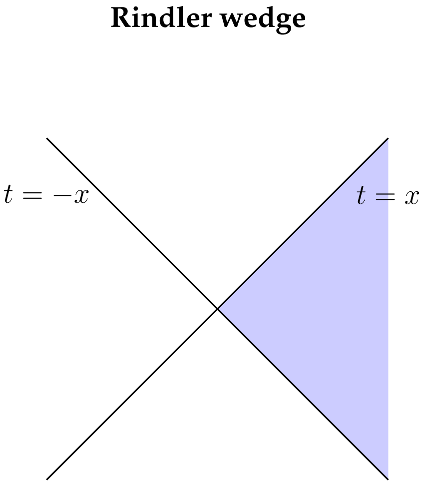
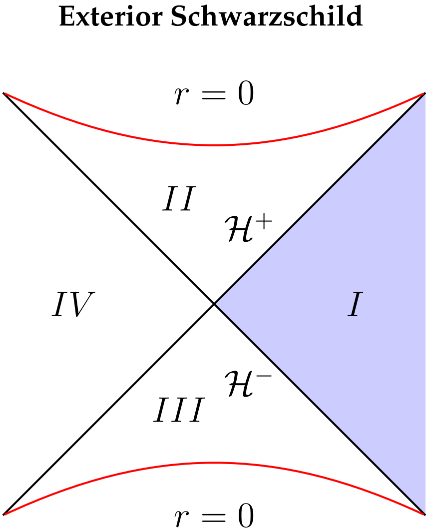
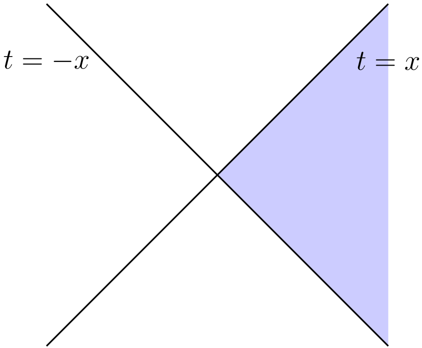


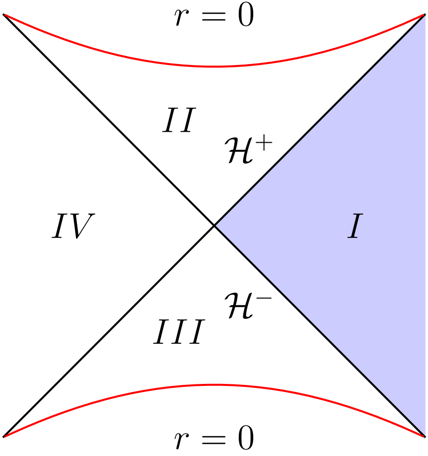



# An analogy [ Kay *CMP* **100** 57 (1985) ]

**Rindler vacuum**

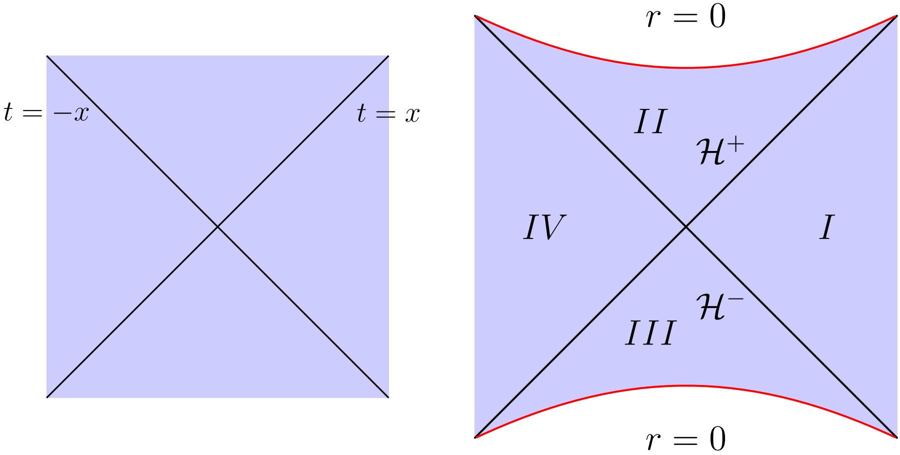



**Boulware state**



# An analogy [ Kay *CMP* **100** 57 (1985) ]

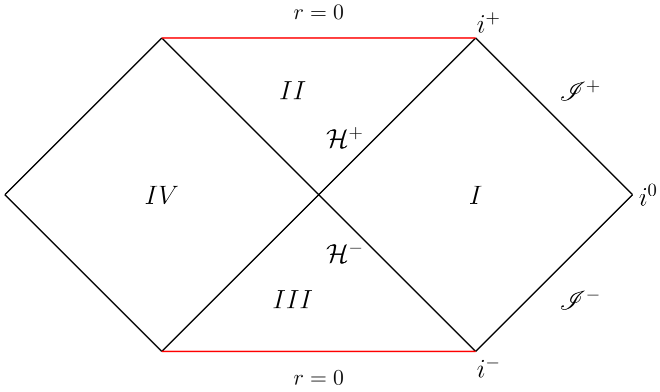
**Minkowski vacuum** **Hartle-Hawking-Israel state**



# HHI state on Schwarzschild space-time

# Schwarzschild space-time

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$$



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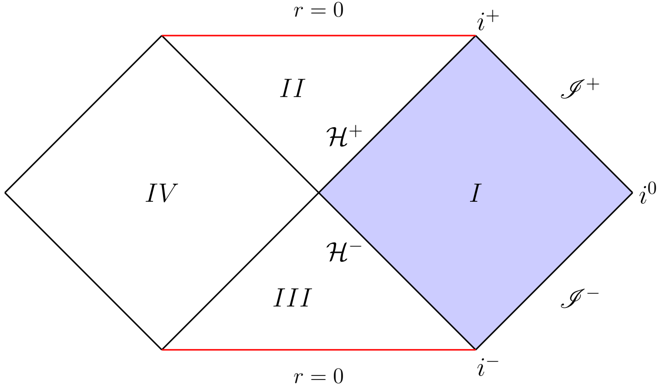


# Canonical quantization of a massless scalar field

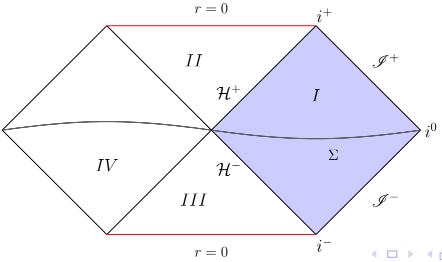
## Klein-Gordon equation

$$\Box\Phi = 0$$

## Klein-Gordon inner product

$$(\Phi_1, \Phi_2)_{KG} = i \int_\Sigma \left[\Phi_2^* \nabla_\mu \Phi_1 - \Phi_1 \nabla_\mu \Phi_2^*\right] \, d\Sigma^\mu$$

Involves time derivative of $\Phi$



# Mode expansion of the massless scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\Phi = \sum_j a_j \phi_j^+ + a_j^\dagger \phi_j^-$$

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## Positive frequency modes

$$\phi_j^+ \propto e^{-i\omega t} \qquad \omega > 0$$

Positive KG “norm”

$$\left( \phi_j^+ , \phi_k^+ \right)_{KG} \propto \delta_{jk},$$

# Mode expansion of the massless scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\Phi = \sum_j a_j \phi_j^+ + a_j^\dagger \phi_j^-$$

## Negative frequency modes

$$\phi_j^- \propto e^{-i\omega t} \qquad \omega < 0$$

Negative KG "norm"

$$\left(\phi_j^-, \phi_k^-\right)_{KG} \propto -\delta_{jk},$$

# Mode expansion of the massless scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\hat{\Phi} = \sum_j \hat{a}_j \phi_j^+ + \hat{a}_j^\dagger \phi_j^-$$

Promote expansion coefficients to operators $\hat{a}_j$, $\hat{a}_j^\dagger$ with

$$\left[\hat{a}_j, \hat{a}_k^\dagger\right] = \delta_{jk} \qquad \left[\hat{a}_j, \hat{a}_k\right] = 0 \qquad \left[\hat{a}_j^\dagger, \hat{a}_k^\dagger\right] = 0$$

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Define the vacuum state $|0\rangle$

$$\hat{a}_j |0\rangle = 0$$

# Massless scalar field modes

$$\phi_{\omega\ell m}(t,r,\theta,\varphi) = \frac{1}{r\mathcal{N}\sqrt{|\omega|}} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

$Y_{\ell m}(\theta)$: spherical harmonics

$\mathcal{N}$: normalization constant independent of $\omega$

Positive frequency with respect to Schwarzschild time $t$: $\omega > 0$

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## Radial mode equation

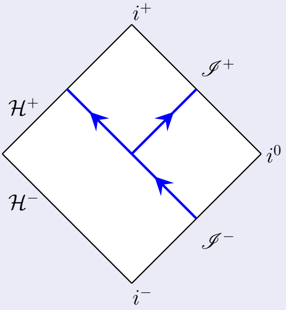
$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell m}(r) \right] R_{\omega\ell}(r) \qquad \frac{dr_*}{dr} = \left( 1 - \frac{2M}{r} \right)^{-1}$$

$$V_{\omega\ell m}(r) = \begin{cases} \omega^2 & \text{as } r \to 2M,\ r_* \to -\infty \\ \omega^2 & \text{as } r \to \infty,\ r_* \to \infty \end{cases}$$

# “In” and “Up” modes

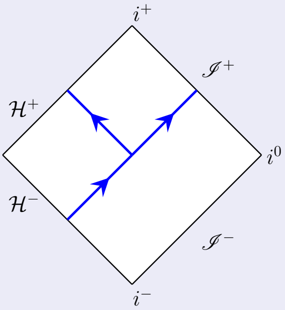
## “In” modes $R^{\mathrm{in}}_{\omega\ell}$

$$\begin{cases} B^{\mathrm{in}}_{\omega\ell} e^{-i\omega r_*} & r_* \to -\infty \\ e^{-i\omega r_*} + A^{\mathrm{in}}_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$$




## “Up” modes $R^{\mathrm{up}}_{\omega\ell}$

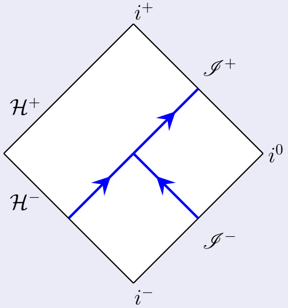
$$\begin{cases} e^{i\omega r_*} + A^{\mathrm{up}}_{\omega\ell} e^{-i\omega r_*} & r_* \to -\infty \\ B^{\mathrm{up}}_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$$



# "Out" and "Down" modes

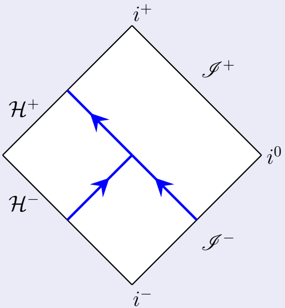
## "Out" modes $R^{\rm out}_{\omega\ell}$

$$\begin{cases} B^{\rm out}_{\omega\ell} e^{i\omega r_*} & r_* \to -\infty \\ e^{i\omega r_*} + A^{\rm out}_{\omega\ell} e^{-i\omega r_*} & r_* \to \infty \end{cases}$$



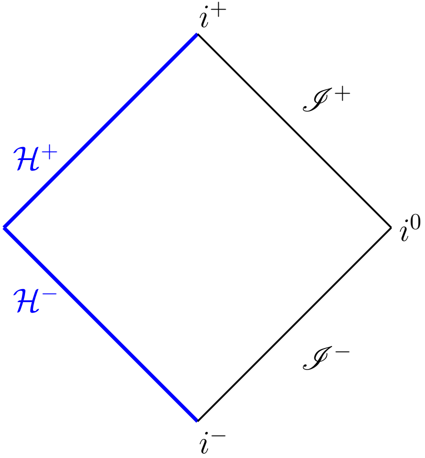

## "Down" modes $R^{\rm down}_{\omega\ell}$

$$\begin{cases} e^{-i\omega r_*} + A^{\rm down}_{\omega\ell} e^{i\omega r_*} & r_* \to -\infty \\ B^{\rm down}_{\omega\ell} e^{-i\omega r_*} & r_* \to \infty \end{cases}$$



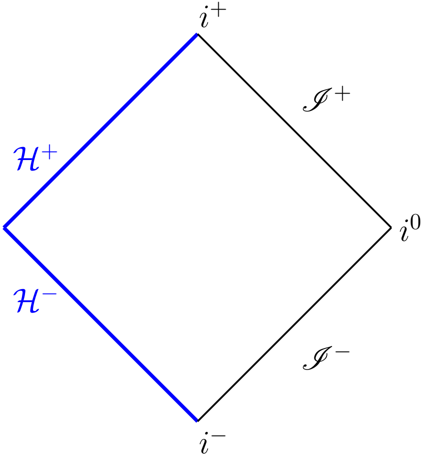
# HHI state [ Hartle & Hawking *PRD* **13** 2188 (1976), Israel *PLA* **57** 107 (1976) ]

- Define positive frequency with respect to Kruskal time $T$



# HHI state [ Hartle & Hawking *PRD* **13** 2188 (1976), Israel *PLA* **57** 107 (1976) ]

- Define positive frequency with respect to Kruskal time $T$
- Use "up" and "down" modes?



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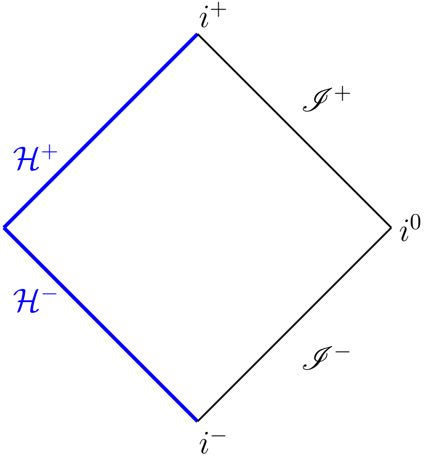
- Define positive frequency with respect to Kruskal time $T$
- Use "up" and "down" modes?
- "Up" and "down" modes are not orthogonal



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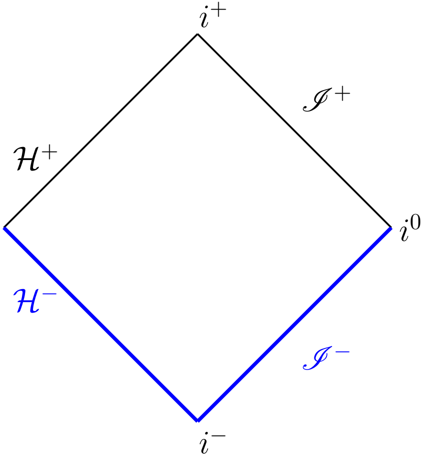
- "Up" and "down" modes are not orthogonal
- Instead use "in" and "up" modes



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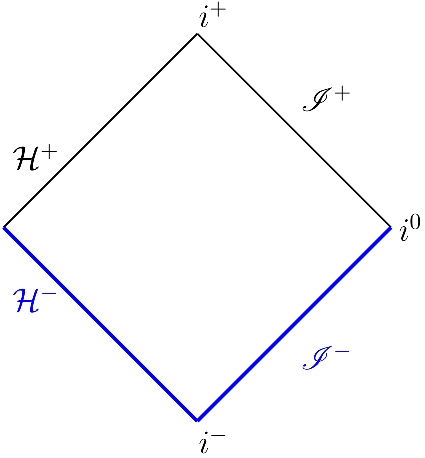
- Define positive frequency with respect to Kruskal time $T$
- Use "up" and "down" modes?
- "Up" and "down" modes are not orthogonal
- Instead use "in" and "up" modes
- Resulting vacuum state is HHI state $|H\rangle$



# Expectation values in the HHI-state $|H\rangle$

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Stress-energy tensor operator

$$\hat{T}_{\mu\nu} = \frac{2}{3}\nabla_\mu\hat{\Phi}\nabla_\nu\hat{\Phi} - \frac{1}{3}\hat{\Phi}\nabla_\mu\nabla_\nu\hat{\Phi} - \frac{1}{6}g_{\mu\nu}\nabla_\lambda\hat{\Phi}\nabla^\lambda\hat{\Phi}$$

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## Unrenormalized stress-energy tensor expectation value

$$\langle H | \hat{T}_{\mu\nu} | H \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^\infty d\omega \coth\left(\frac{\omega}{2T_H}\right) \left\{ T_{\mu\nu}\left[\phi^{\text{in}}_{\omega\ell m}\right] + T_{\mu\nu}\left[\phi^{\text{up}}_{\omega\ell m}\right] \right\}$$

[ Candelas *PRD* **21** 2185 (1980) ]

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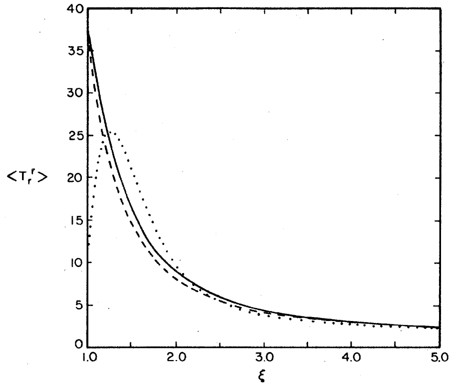
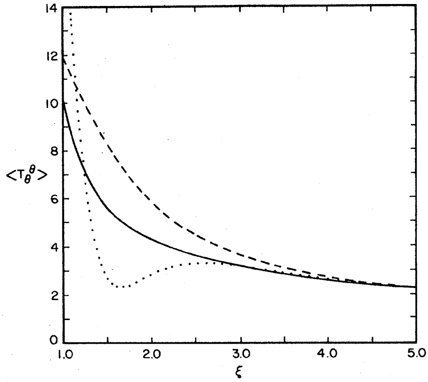
[ Candelas *PRD* **21** 2185 (1980) ]

Compute renormalized expectation values using point-splitting

[ Howard *PRD* **30** 2532 (1984) ]

# $\langle H|\hat{T}_{\mu\nu}|H\rangle$ for a massless scalar field

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# HHI state $|H\rangle$

For a quantum scalar field on Schwarzschild space-time

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## Properties

- Thermal state in region $I$
- Regular on the horizons
- $\langle H|\hat{T}_{\mu\nu}|H\rangle$ finite everywhere in region $I$
- Time-reversal symmetric

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For a quantum scalar field on Schwarzschild space-time

## Properties

- Thermal state in region $I$
- Regular on the horizons
- $\langle H|\hat{T}_{\mu\nu}|H\rangle$ finite everywhere in region $I$
- Time-reversal symmetric

## Rigorous results on the existence of $|H\rangle$

Kay *CMP* **100** 57 (1985) HHI state in regions $I$ & $IV$

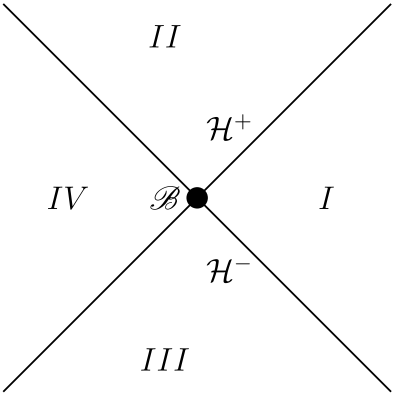
Jacobson *PRD* **50** R6031 (1994) HHI state on Euclidean section

Sanders *IJMPA* **28** 1330010 (2013) HHI state on Kruskal space-time

Sanders *Lett. Math. Phys.* **105** 575 (2015) HHI state across horizons

# The Kay-Wald theorem [ Kay & Wald *Phys. Rept.* **207** 49 (1991) ]

Globally hyperbolic space-time with a bifurcate Killing horizon




Wedge isometry maps $I \leftrightarrow IV$

[ Kay *JMP* **34** 4519 (1993), Kay & Lupo *CQG* **33** 215001 (2016) ]

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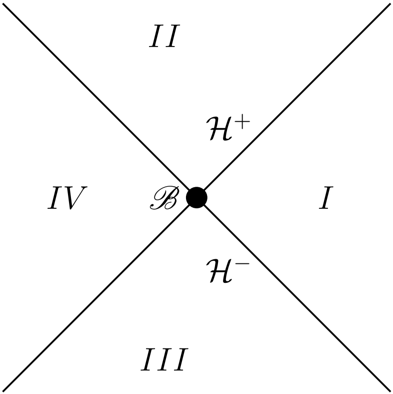
### Theorem

Globally hyperbolic space-time with a bifurcate Killing horizon

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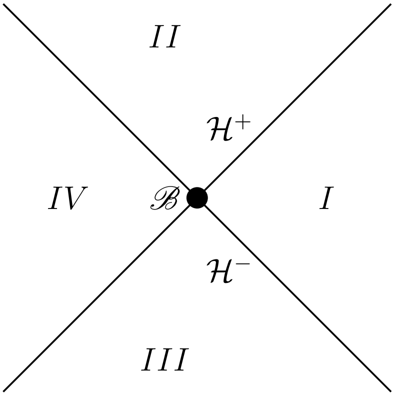
**Theorem**

- On a large subalgebra of observables, there can be at most one quasifree, isometry invariant, Hadamard state

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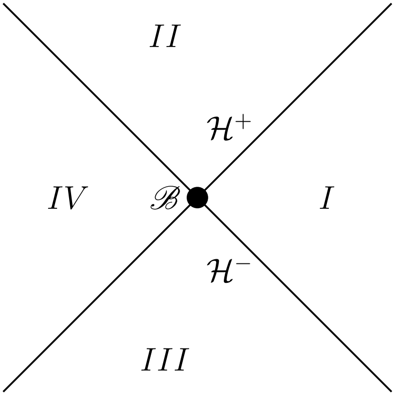
Globally hyperbolic space-time with a bifurcate Killing horizon

### Theorem

- On a large subalgebra of observables, there can be at most one quasifree, isometry invariant, Hadamard state
- This state, if it exists, is a KMS state at the Hawking temperature $T_H$ on observables in the subalgebra localized in region $I$

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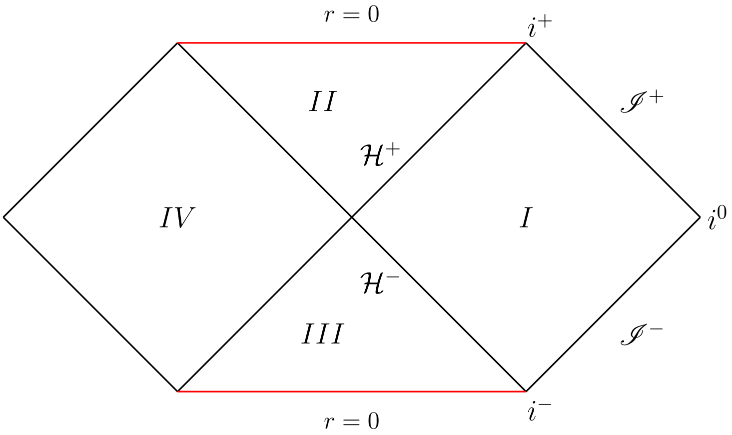
# HHI state $|H\rangle$ on Schwarzschild

## The Kay-Wald theorem [ Kay & Wald *Phys. Rept.* **207** 49 (1991) ]

$|H\rangle$ exists and is unique on Schwarzschild



# Massless fermion field $\Psi$

## Dirac equation

$$\gamma^{\mu}\nabla_{\mu}\Psi = 0$$

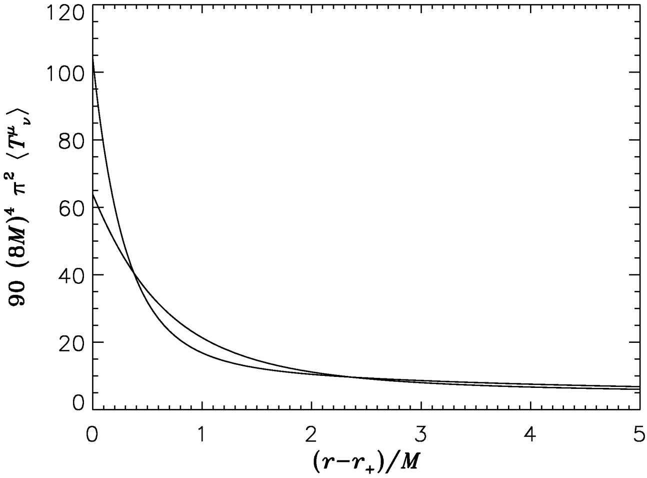
## Canonical quantization

- Expansion of classical field in orthonormal basis of field modes
- "in" and "up" modes
- Positive frequency with respect to Kruskal time $T$

## Stress-energy tensor

$$\hat{T}_{\mu\nu} = \frac{i}{8}\left\{\left[\hat{\overline{\Psi}}, \gamma_{\mu}\nabla_{\nu}\hat{\Psi}\right] + \left[\hat{\overline{\Psi}}, \gamma_{\nu}\nabla_{\mu}\hat{\Psi}\right] - \left[\nabla_{\mu}\hat{\overline{\Psi}}, \gamma_{\nu}\hat{\Psi}\right] - \left[\nabla_{\nu}\hat{\overline{\Psi}}, \gamma_{\mu}\hat{\Psi}\right]\right\}$$

# $\langle H|\hat{T}_{\mu\nu}|H\rangle$ for a massless fermion field $\Psi$




[ Carlson et al *PRL* **91** 051301 (2003) ]

# Kerr space-time

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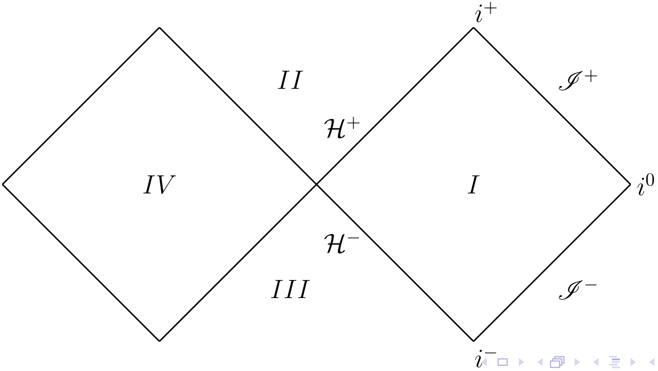
$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - a\sin^2\theta\, d\varphi\right]^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left[\left(r^2+a^2\right)d\varphi - a dt\right]^2$$

$$\Delta = r^2 - 2Mr + a^2 \qquad \Sigma = r^2 + a^2\cos^2\theta$$



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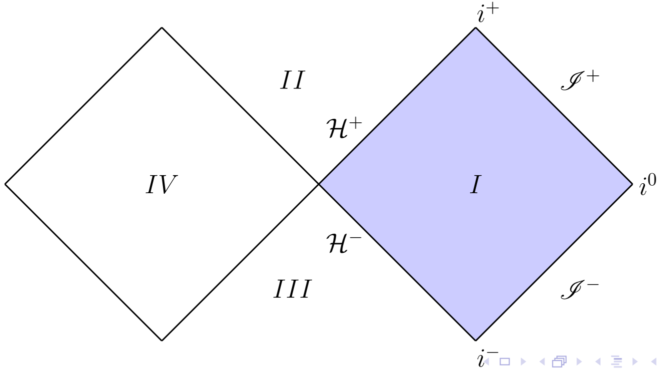

# Features of Kerr space-time

## Event horizon

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## Stationary limit surface

$$r_S = M + \sqrt{M^2 - a^2 \cos^2 \theta}$$

For $r_H < r < r_S$ an observer cannot remain at rest relative to infinity and must have a non-zero angular velocity

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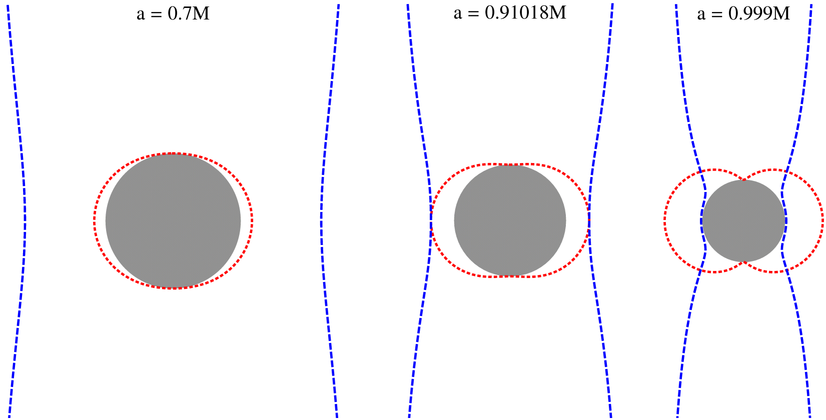
For $r_H < r < r_S$ an observer cannot remain at rest relative to infinity and must have a non-zero angular velocity

## Speed-of-light surface

An observer can have the same angular velocity as the event horizon between $r = r_H$ and the speed-of-light surface $\mathscr{S}_L$

# Location of stationary limit surface and speed-of-light surface




[ Casals et al *PRD* **87** 064027 (2013) ]

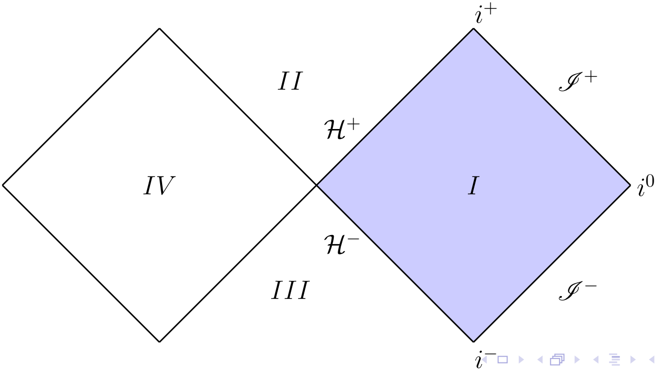
# HHI state on Kerr space-time

# Quantum scalar field

# The Kay-Wald theorem [ Kay & Wald *Phys. Rept.* **207** 49 (1991) ]

## Properties of $|H\rangle$ on Schwarzschild

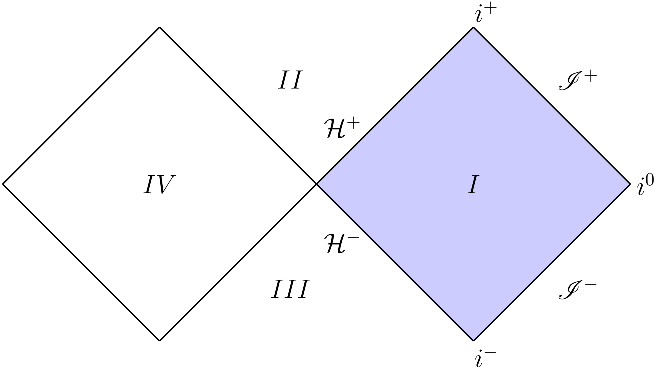
- Regular on and outside horizon
- Time-reversal symmetric
- Thermal state in region $I$



# The Kay-Wald theorem [ Kay & Wald *Phys. Rept.* **207** 49 (1991) ]

## Theorem

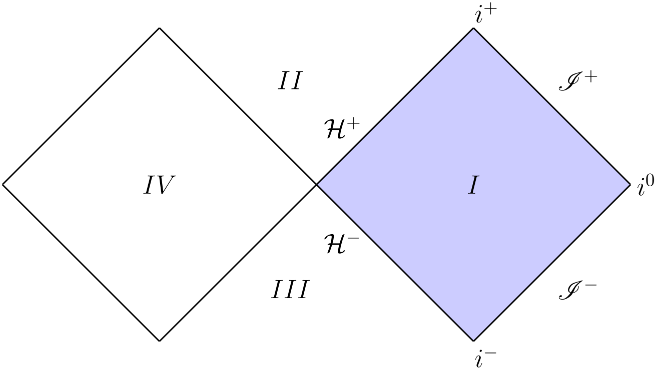
There does not exist any Hadamard state on Kerr which is invariant under the isometries generating the event horizon



# The Kay-Wald theorem [ Kay & Wald *Phys. Rept.* **207** 49 (1991) ]

## Theorem

No HHI state exists for a quantum scalar field on Kerr



# Massless scalar field on Kerr space-time

## Scalar field modes

$$\phi_{\omega\ell m}(t,r,\theta,\varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2+a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos\theta) R_{\omega\ell m}(r)$$

$S_{\omega\ell m}(\cos\theta)$: spheroidal harmonics

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$S_{\omega\ell m}(\cos\theta)$: spheroidal harmonics

## Radial mode equation

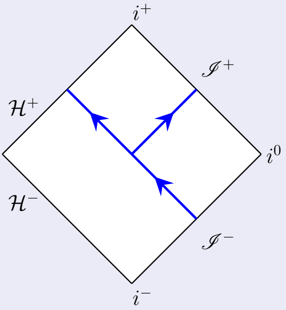
$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell m}(r) \right] R_{\omega\ell m}(r) \qquad \frac{dr_*}{dr} = \frac{r^2+a^2}{\Delta}$$

$$V_{\omega\ell m}(r) = \begin{cases} \widetilde{\omega}^2 = (\omega - m\Omega_H)^2 & \text{as } r \to r_H, r_* \to -\infty \\ \omega^2 & \text{as } r \to \infty, r_* \to \infty \end{cases}$$

# "In" and "Up" modes

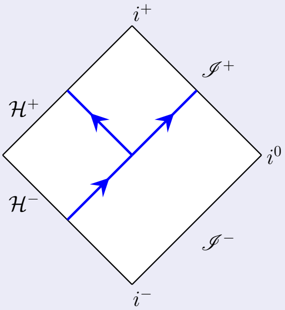
## "In" modes $R^{\mathrm{in}}_{\omega\ell m}$

$$\begin{cases} B^{\mathrm{in}}_{\omega\ell m} e^{-i\tilde{\omega} r_*} & r_* \to -\infty \\ e^{-i\omega r_*} + A^{\mathrm{in}}_{\omega\ell m} e^{i\omega r_*} & r_* \to \infty \end{cases}$$




## "Up" modes $R^{\mathrm{up}}_{\omega\ell m}$

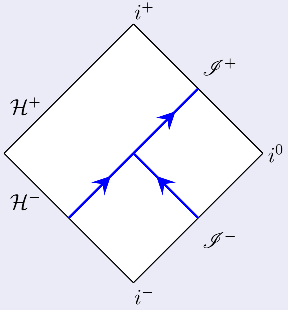
$$\begin{cases} e^{i\tilde{\omega} r_*} + A^{\mathrm{up}}_{\omega\ell m} e^{-i\tilde{\omega} r_*} & r_* \to -\infty \\ B^{\mathrm{up}}_{\omega\ell m} e^{i\omega r_*} & r_* \to \infty \end{cases}$$



# "Out" and "Down" modes

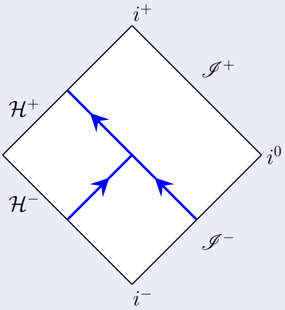
## "Out" modes $R^{\text{out}}_{\omega\ell}$

$$\begin{cases} B^{\text{out}}_{\omega\ell} e^{i\tilde{\omega} r_*} & r_* \to -\infty \\ e^{i\omega r_*} + A^{\text{out}}_{\omega\ell} e^{-i\omega r_*} & r_* \to \infty \end{cases}$$




## "Down" modes $R^{\text{down}}_{\omega\ell}$

$$\begin{cases} e^{-i\tilde{\omega} r_*} + A^{\text{down}}_{\omega\ell} e^{i\tilde{\omega} r_*} & r_* \to -\infty \\ B^{\text{down}}_{\omega\ell} e^{-i\omega r_*} & r_* \to \infty \end{cases}$$



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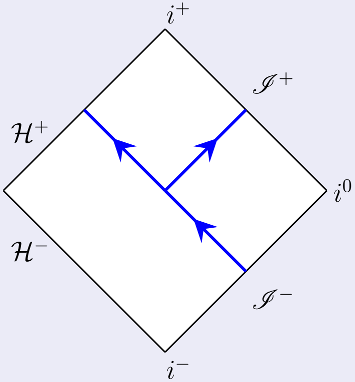


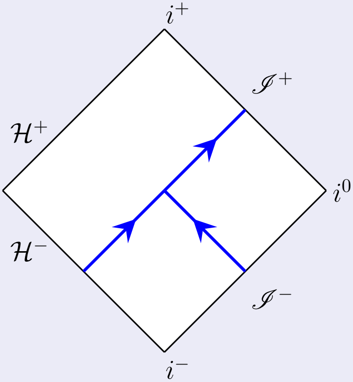

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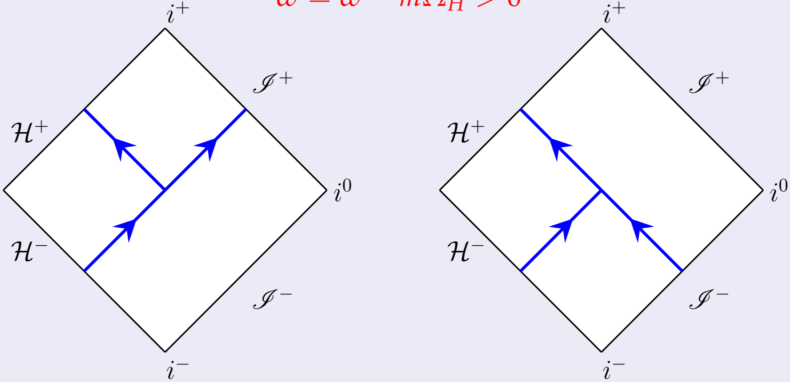
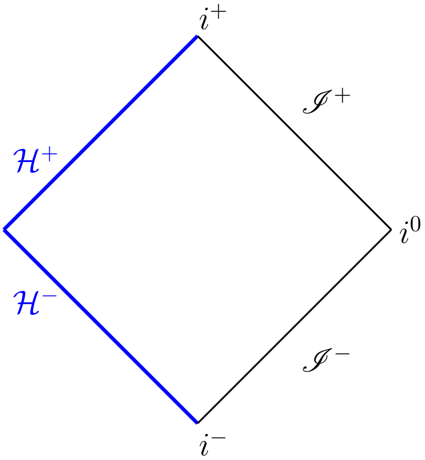


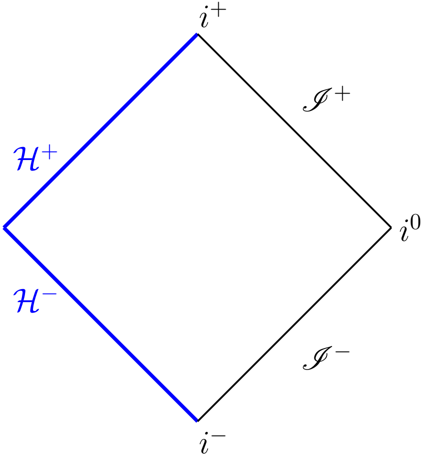
# A HHI-like state for a scalar field on Kerr?

- Define positive frequency with respect to Kruskal time $T$



# A HHI-like state for a scalar field on Kerr?

- Define positive frequency with respect to Kruskal time $T$
- Use “up” and “down” modes with $\widetilde{\omega} > 0$?



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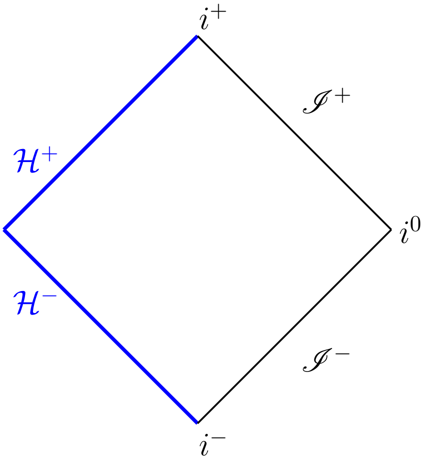
- "Up" and "down" modes are not orthogonal



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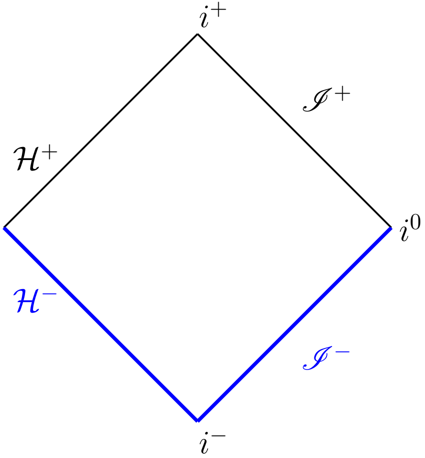

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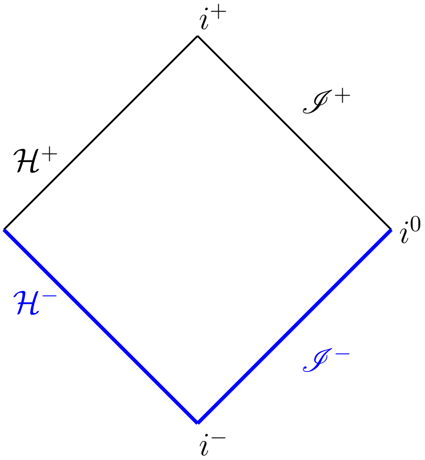
- Define positive frequency with respect to Kruskal time $T$
- Use "up" and "down" modes with $\widetilde{\omega} > 0$?
- "Up" and "down" modes are not orthogonal
- Instead use "in" and "up" modes
- But "in" modes have positive "norm" for $\omega > 0$



# Attempts at defining a HHI-like state for Kerr

## $|CCH\rangle$ [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\langle CCH|\hat{T}_{\mu\nu}|CCH\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^{\infty} d\omega \coth\left(\frac{\omega}{2T_H}\right) T_{\mu\nu}\left[\phi^{\text{in}}_{\omega\ell m}\right] + \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^{\infty} d\widetilde{\omega} \coth\left(\frac{\widetilde{\omega}}{2T_H}\right) T_{\mu\nu}\left[\phi^{\text{up}}_{\omega\ell m}\right]$$

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- Does not represent an equilibrium state
- Regular outside the event horizon

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- Method for computing renormalized expectation values on Kerr has been elusive until recently
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**"Past" Boulware state $|B^{-}\rangle$** [ Unruh *PRD* **10** 3194 (1974) ]

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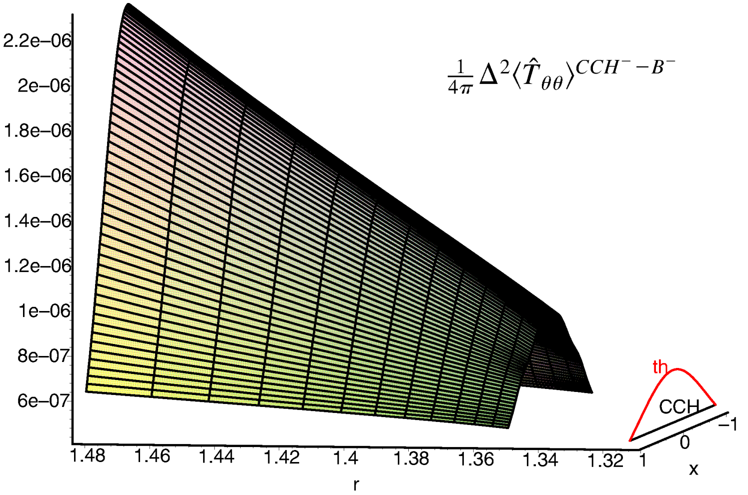
- Diverges on the event horizon
- Regular everywhere outside the event horizon in region $I$

# $\langle CCH|\hat{T}_{\mu\nu}|CCH\rangle$ for an electromagnetic field




[ Casals & Ottewill *PRD* **71** 124016 (2005) ]

# Attempts at defining a Hartle-Hawking state for Kerr

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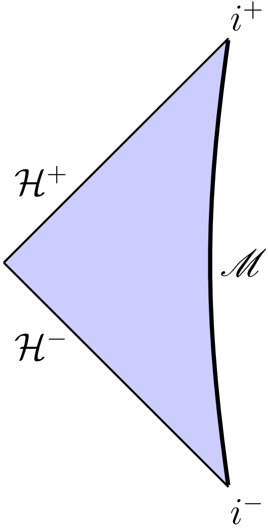
- Potentially an equilibrium state
- Divergent everywhere except on the axis of rotation

[ Ottewill & Winstanley *PRD* **62** 084018 (2000) ]

# Kerr space-time with a mirror

Mirror $\mathscr{M}$ at fixed $r = r_0$ inside $\mathscr{S}_L$




[ Duffy & Ottewill *PRD* **77** 024007 (2008) ]

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## Modes

$$\phi^{\mathscr{M}}_{\omega\ell m} = \begin{cases} \phi^{\text{up}}_{\omega\ell m} - \dfrac{R^{\text{up}}_{\omega\ell m}(r_0)}{R^{\text{in}}_{\omega\ell m}(r_0)}\phi^{\text{in}}_{\omega\ell m} & \omega > 0 \\ \phi^{\text{up}}_{\omega\ell m} - \dfrac{R^{\text{up}}_{\omega\ell m}(r_0)}{R^{\text{in}*}_{-\omega\ell -m}(r_0)}\phi^{\text{in}*}_{-\omega\ell -m} & \omega < 0 \end{cases}$$

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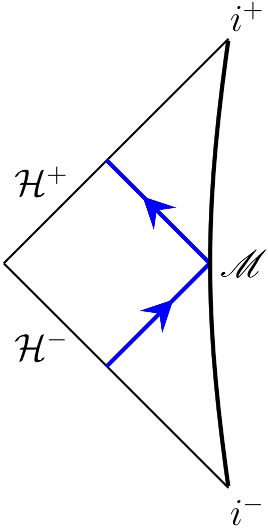

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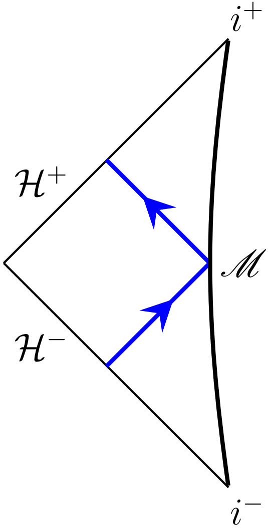
Positive KG "norm" for $\widetilde{\omega} > 0$

[ Duffy & Ottewill *PRD* **77** 024007 (2008) ]



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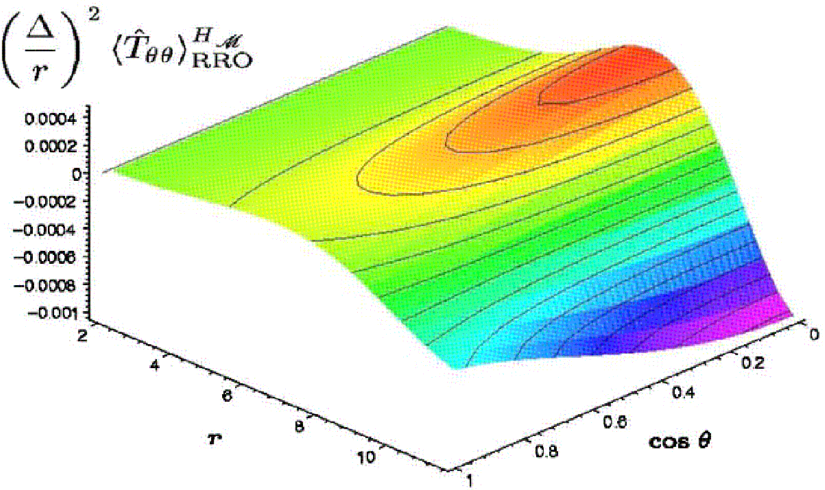
- Compute expectation values relative to $|B_{\mathscr{M}}\rangle$
- $|B_{\mathscr{M}}\rangle$ defined by taking modes to have positive frequency with respect to $t$
- $|B_{\mathscr{M}}\rangle$ diverges on $\mathcal{H}^{\pm}$

# $\langle H_{\mathscr{M}} | \hat{T}_{\mu\nu} | H_{\mathscr{M}} \rangle$

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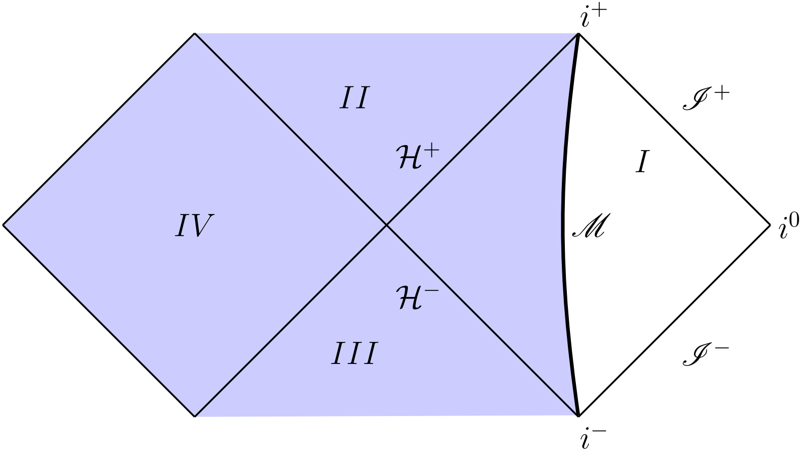
# HHI-states on space-times with enclosed horizons

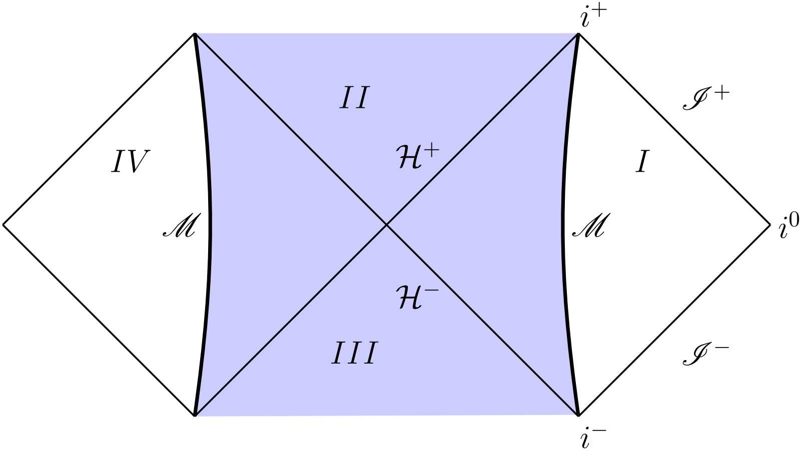
Non-existence of HHI-state on Kruskal space-time with a single mirror




[ Kay & Lupo *CQG* **33** 215001 (2016) ]

# HHI-states on space-times with enclosed horizons

## Existence of HHI-state on Kruskal space-time with two mirrors




[ Kay *GRG* **47** 31 (2015) ]

# HHI state on Kerr space-time

# Quantum fermion field

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## Dirac equation

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## Positivity of the Dirac norm

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- All modes have positive Dirac norm
- Both positive frequency and negative frequency modes have positive Dirac norm
- More freedom in the choice of positive frequency?

# Mode expansion of the massless fermion field $\Psi$

Expand classical field in terms of orthonormal basis of field modes

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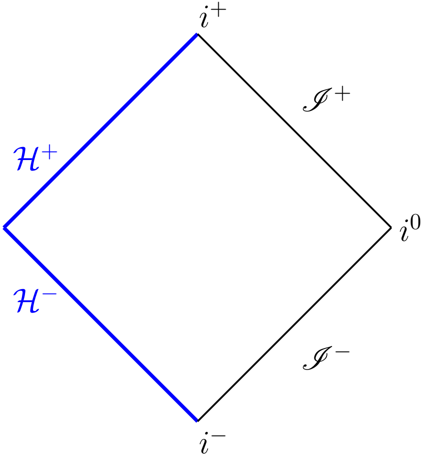
Define the vacuum state $|0\rangle$

$$\hat{b}_j |0\rangle = 0 = \hat{c}_j |0\rangle$$

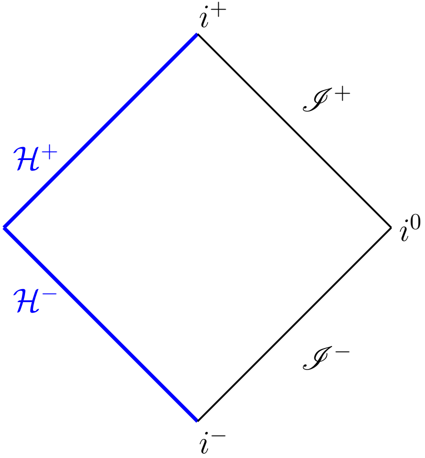
# A HHI-like state for a fermion field on Kerr?

- Define positive frequency with respect to Kruskal time $T$



# A HHI-like state for a fermion field on Kerr?

- Define positive frequency with respect to Kruskal time $T$
- Use "up" and "down" modes with $\widetilde{\omega} > 0$?



# A HHI-like state for a fermion field on Kerr?

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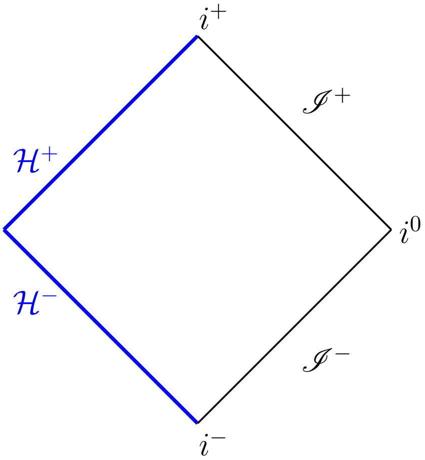

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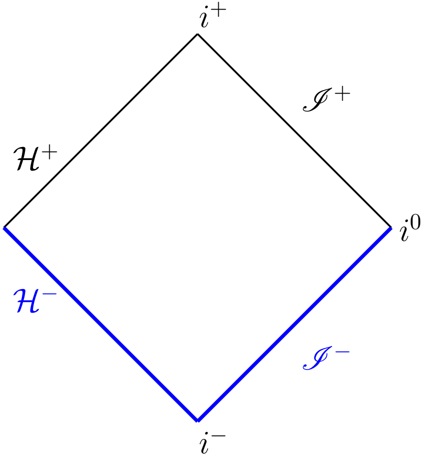

# A HHI-like state for a fermion field on Kerr?

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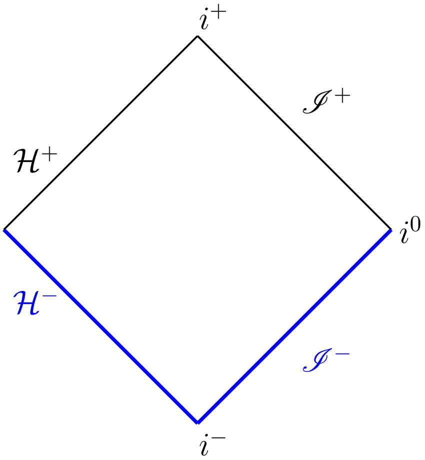
- Use "up" and "down" modes with $\widetilde{\omega} > 0$?
- "Up" and "down" modes are not orthogonal
- Instead use "in" and "up" modes with $\widetilde{\omega} > 0$
- Call the resulting vacuum state $|H\rangle$



# Unrenormalized expectation values

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$|H\rangle$ [ Casals et al *PRD* **87** 064027 (2013) ]

$$\langle H|\hat{T}_{\mu\nu}|H\rangle = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^{\infty} d\widetilde{\omega}\,\tanh\left(\frac{\widetilde{\omega}}{2T_H}\right)\left\{T_{\mu\nu}\left[\psi^{\text{in}}_{\omega\ell m}\right]+T_{\mu\nu}\left[\psi^{\text{up}}_{\omega\ell m}\right]\right\}$$

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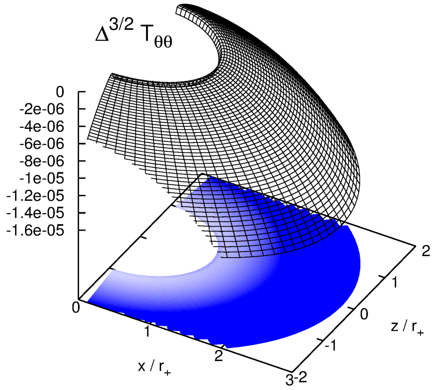
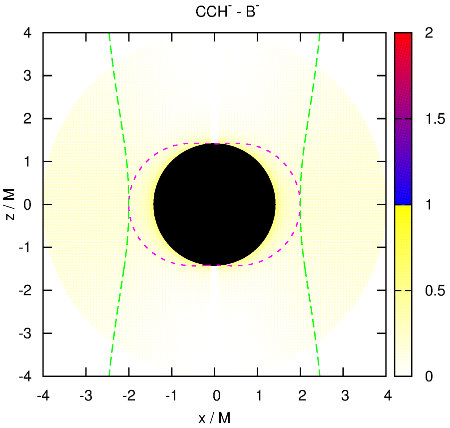
$|CCH\rangle$ [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned}\langle CCH|\hat{T}_{\mu\nu}|CCH\rangle &= \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^{\infty} d\omega\,\tanh\left(\frac{\omega}{2T_H}\right)T_{\mu\nu}\left[\psi^{\text{in}}_{\omega\ell m}\right]\\ &\quad+\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\int_0^{\infty} d\widetilde{\omega}\,\tanh\left(\frac{\widetilde{\omega}}{2T_H}\right)T_{\mu\nu}\left[\psi^{\text{up}}_{\omega\ell m}\right]\end{aligned}$$

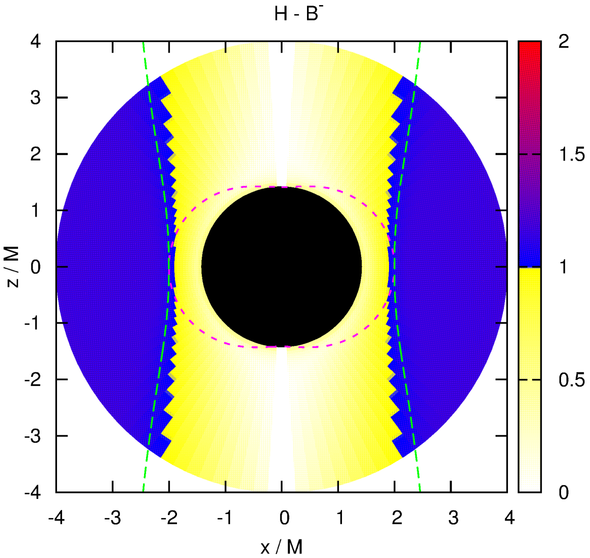
# $\langle CCH|\hat{T}_{\mu\nu}|CCH\rangle$ for a fermion field




[ Casals et al *PRD* **87** 064027 (2013) ]

# $\langle H|\hat{T}_{\mu\nu}|H\rangle$ for a fermion field [ Casals et al *PRD* **87** 064027 (2013) ]



# HHI states on black hole space-times

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## Schwarzschild

Kay *CMP* **100** 57 (1985) Existence of HHI state

Kay & Wald *Phys. Rept.* **207** 49 (1991) Uniqueness of HHI state

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- Nonequilibrium state
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## HHI-like states for fermions on Kerr

- Equilibrium state diverges on and outside $\mathscr{S}_L$
- Kay-Wald theorem extends to fermions?