Applications of local gauge covariance: Anomalies and QED in external potentials

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DPG Frühjahrstagung
Berlin, March 2015

This talk is based on

- arXiv:1210.4031 [Rev. Math. Phys. 26 (2014) 1330012] (Local gauge covariance)
- arXiv:1311.7661 (Background independence)
- arXiv:1407.1994 [Nucl. Phys. B 890 (2015) 1] (Anomalies)
- arXiv:1501.05912 [with J. Schlemmer] (QED in external potentials)
- arXiv:1501.06527 (Time-dependent Schwinger effect)

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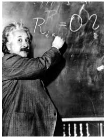

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Here, I present a generalization of the framework of local covariance to encompass also external gauge fields. As applications, I discuss the computation of anomalies and some results in QED in external potentials. The guiding theme is the definition of the current

$$j_I^\mu = \bar{\psi}\gamma^\mu T_I \psi$$

in a locally gauge covariant way by a point-splitting w.r.t. the Hadamard parametrix.

Outline

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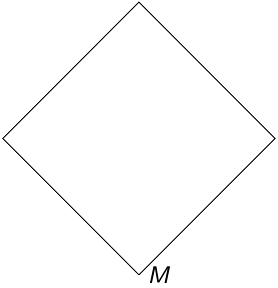
Local covariance



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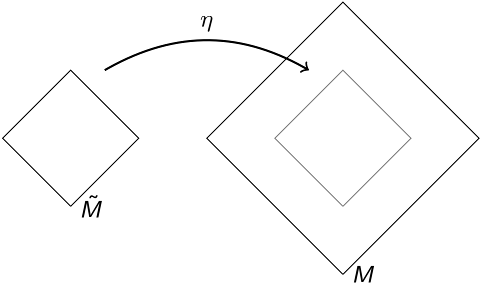

- To each globally hyperbolic spacetime M, associate the algebra $\mathfrak{A}(M)$ of observables measurable there.
- For each isometric embedding $\eta : \tilde{M} \to M$ there is an injective homomorphism $\alpha_\eta : \mathfrak{A}(\tilde{M}) \to \mathfrak{A}(M)$, such that $\alpha_\eta \circ \alpha_{\eta'} = \alpha_{\eta \circ \eta'}$ and $\alpha_{\mathrm{id}_M} = \mathrm{id}_{\mathfrak{A}(M)}$.

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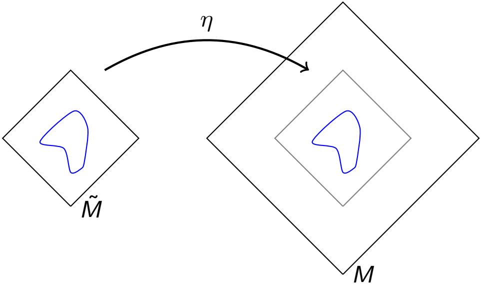

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- A field Φ associates to each M a linear map $\mathfrak{Test}_c(M) \to \mathfrak{A}(M)$, such that $\alpha_\eta(\Phi_{\tilde{M}}(t)) = \Phi_M(\eta_* t)$. This implies that $\Phi_M(x)$ can only depend on the jet of the geometric data at x. Examples are
  - The linear field $\phi(x)$.
  - The stress-energy tensor $T_{\mu\nu}(x)$.

Local gauge covariance

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- For each bundle morphism $\eta : \tilde{P} \to P$, covering an isometric embedding $\chi : \tilde{M} \to M$ and such that $\eta^* A = \tilde{A}$, there is an injective homomorphism $\alpha_\eta : \mathfrak{A}(\tilde{G}) \to \mathfrak{A}(G)$, such that $\alpha_\eta \circ \alpha_{\eta'} = \alpha_{\eta \circ \eta'}$ and $\alpha_{\mathrm{id}_G} = \mathrm{id}_{\mathfrak{A}(G)}$.
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  - The current $j^\mu(x)$.

Outline

A two-point function

$$w(x, y) = \omega(\phi(x)\phi(y))$$

which locally has the same singularities as the vacuum two-point function is called a Hadamard two-point function (this can be elegantly formulated using the wave front set [Radzikowski 96]). There is very good reason to assume that physically sensible states have Hadamard two-point functions:

- A Hadamard state always exists [Fulling, Narcowich & Wald 81; ...]
- The ground state in static situations is Hadamard [Sahlmann & Verch 00, Wrochna 12]
- A two-point function that is Hadamard in a neighborhood of a Cauchy surface is Hadamard everywhere [Fulling, Sweeny & Wald 78; Sahlmann & Verch 01; Sanders 10]
- In Hadamard states, Wick powers (stress-energy tensor, current) have finite fluctuations [Brunetti & Fredenhagen 00]

A further important fact is that Hadamard two-point functions differ only by smooth functions [Sanders 10].

Leaving aside the question of how to define interacting fields, let us concentrate on the definition of non-linear fields (Wick powers) for a free field theory in a given background, i.e., ignoring backreaction. In particular, we are interested in the current

$$j_I^\mu = \bar{\psi} T_I \gamma^\mu \psi.$$

Defining the point-wise product by a coinciding point limit and evaluating in a Hadamard state, one obtains a divergence. An approach inspired by QFT in the vacuum would be to take a reference two-point function w_0 and define

$$\omega(j_I^\mu(x)) = \lim_{x' \to x} \left(\omega(\bar{\psi}(x) T_I \gamma^\mu \psi(x')) - \operatorname{tr} T_I \gamma^\mu w_0(x, x') \right),$$

i.e., normal ordering. However, there is no locally covariant choice of a two-point function. The way out [Adler, Liebermann & Ng 77; Wald 77; Hollands & Wald 01] is to replace w_0 by the Hadamard parametrix, which is locally and covariantly defined and has the same singularities as a Hadamard two-point function. It is not a solution to the wave equation, which can be seen as the origin of anomalies. It is not unique, but smooth, locally and covariantly constructed terms can be added. This is a reflection of the renormalisation ambiguity of Wick powers.

The parametrix

To construct the parametrix for the Dirac operator $D = \nabla\!\!\!/ + m$, one first defines

$$P = D^2 = \tilde{\nabla}^\mu \tilde{\nabla}_\mu + \tfrac{1}{4} e F_{\lambda\rho}[\gamma^\lambda, \gamma^\rho] - \tfrac{1}{4} R - (d-1) m^2,$$

with $\tilde{\nabla}_\mu = \nabla_\mu + m\gamma_\mu$. Positive/negative frequency parametrices for P are formally given by [Sahlmann & Verch 01]

$$h^\pm(x, x') = \frac{1}{2\pi} \sum_{k=0}^{\infty} V_k(x, x') T_k^\pm(x, x'),$$

where the Hadamard coefficients V_k are smooth sections, locally defined by the geometric data and $T_k^\pm$ are distributions. Explicitly, in even dimension $d = 2m$, they are given by

$$T_k^\pm = \lim_{\varepsilon \to +0} \begin{cases} C'_{k,m} (-\Gamma_{\pm\varepsilon})^{k+1-m} & k+1 < m \\ C_{k,m} \Gamma^{k+1-m} \log(-\Gamma_{\pm\varepsilon})/\Lambda^2 & k+1 \geq m \end{cases}$$

where Γ_ε is the squared geodesic distance equipped with some $i\varepsilon$ prescription and Λ is a length scale. The parametrices for D are given by

$$H^\pm(x', x) = \tfrac{1}{2}(D + D^{*\prime}) h^\pm(x, x').$$

This is the analog of the construction of propagators for D [Dimock 82; Mühlhoff 11]:

$$S^{\mathrm{ret/adv}} = D \circ \Delta^{\mathrm{ret/adv}} = \Delta^{\mathrm{ret/adv}} \circ D.$$

Outline

The divergence of the current

Classically, the divergence $\nabla_\mu j_I^\mu$ of the current $j^\mu = \mathrm{tr}\, \bar{\psi} T_I \gamma^\mu \psi$ vanishes on-shell. That this also holds at the quantum level is not guaranteed. An obstruction to fulfill this constitutes an anomaly.

In the present setting, anomalies occur, because the parametrix is a bi-solution only modulo smooth remainders. An anomaly can then be understood as an obstruction to finding a locally and covariantly defined parametrix H such that

$$\mathrm{tr}[QH] = 0,$$

where Q is a bi-differential operator which vanishes on bi-solutions, and the square brackets denote the coinciding point limit. Thus, the trace anomaly for scalar [Moretti 03; Hollands & Wald 05] and Dirac fermions [Dappiaggi, Hack & Pinamonti 09] was computed.

In the present framework, we can calculate the chiral anomalies in an elementary way, without recourse to Riemannian spaces, ill-defined objects (loop integrals ...), or fancy mathematics (index theory, extension of distributions).

Absence of anomalies for Dirac fermions & renormalization ambiguities

Consider Dirac fermions. For the divergence of the current, we have

$$\nabla_\mu j^\mu = \bar{\psi} T_I \not{\nabla} \psi + \not{\nabla} \bar{\psi} T_I \psi = \bar{\psi} T_I D \psi - D^* \bar{\psi} T_I \psi$$

so classically the divergence of the current vanishes on-shell. On the quantum level, we have to consider

$$\begin{aligned} \operatorname{tr} T_I[(D' - D^*)H] &= \tfrac{1}{2} \operatorname{tr} T_I[(D' - D^*)(D' + D^*)h] \\ &= \tfrac{1}{2} \operatorname{tr} T_I \gamma^\mu [P' h - P^* h], \end{aligned}$$

which vanishes [Moretti 03]. Hence, there is no gauge anomaly for Dirac fermions.

Possible redefinitions (finite renormalizations) of the current can only consist in adding locally and covariantly defined vector fields with vanishing divergence. There is only one, namely the external current. This ambiguity corresponds to a charge renormalization.

Anomalies

For left-handed fermions, we obtain (χ is the chirality operator)

$$\nabla_\mu j_I^\mu = \tfrac{\hbar}{2} \operatorname{tr} \left(T_I \left([Ph] + [\not\nabla \not\nabla' h] \right) \chi \right) = \tfrac{\hbar}{2\pi} C_m \operatorname{tr} \left(T_I [V_m] \chi \right).$$

This corresponds to the expression obtained by heat kernel techniques. For $d = 4$,

$$\nabla_\mu j_I^\mu = \tfrac{i}{32\pi^2} \tfrac{1}{\sqrt{-g}} \varepsilon^{\mu\nu\lambda\rho} \operatorname{tr}_V T_I \left(F_{\mu\nu} F_{\lambda\rho} + \tfrac{1}{24} R_{\sigma\xi\mu\nu} R^{\sigma\xi}{}_{\lambda\rho} \right).$$

- Vanishing of $\nabla_\mu j_I^\mu$ implies that there exists a renormalization prescription such that the current is conserved also in the interacting theory (Adler-Bardeen theorem).
- Also the purely gravitational anomaly [Alvarez-Gaumé & Witten 83] can be computed in this framework.

Outline

Renormalization prescriptions for the current

- In Schwinger's prescription [Schwinger 51], one performs the limit of coinciding points in a symmetric way, thus canceling some divergences:

$$j^\mu(x) = \tfrac{e}{2} \lim_{t \to 0} \left(U(x+tv, x) \bar{\psi}(x+tv) \gamma^\mu \psi(x) + U(x, x+tv) \bar{\psi}(x) \gamma^\mu \psi(x+tv) \right) ,$$

  where v is some direction, and $U(x, y)$ the parallel transport along the straight line from y to x.
    - Not all divergences are canceled, unless the external current J^μ vanishes.
    - Even if J^μ vanishes, the result depends on v.
    - Often, the parallel transport U is omitted, leading to further divergences.
- For static external potentials, there is the mode sum formula [Wichmann & Kroll 56]

$$\langle \rho(x) \rangle = \tfrac{e}{2} \left(\sum\nolimits_{+} \bar{\psi}_n(x) \gamma^0 \psi_n(x) - \sum\nolimits_{-} \bar{\psi}_n(x) \gamma^0 \psi_n(x) \right) ,$$

  where $\sum_\pm$ stands for the sum over positive and negative frequency modes.
    - In general, the expression is ill-defined.
    - In situations where one might make sense out of it, it gives the wrong result.

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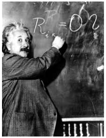

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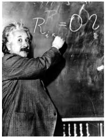

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A distribution [two-point function] R such as occurs in nature according to the above assumption can be divided naturally into two parts

$$R = R_a + R_b,$$

where R_a contains all the singularities and is also completely fixed for any given field, so that any alteration one can make in the distribution of electrons and positrons will correspond to an alteration in R_b but none in R_a.
[...]
It therefore appears reasonable to make the assumption that the electric and current densities corresponding to R_b are those which are physically present, arising from the distribution of electrons and positrons. In this way we can remove the infinities [...].

P.A.M. Dirac, Discussion of the infinite distribution of electrons in the theory of the positron, Proc. Camb. Phil. Soc. 30 (1934) 150.

Explicitly, the relevant contraction of the Hadamard parametrix is given by

$$U(x, x+y)\,\mathrm{tr}\,H(x, x+y)\gamma_\mu = \\ -\frac{1}{2\pi^2}\left\{4i\frac{y_\mu}{y_\varepsilon^4} + im^2\frac{y_\mu}{y_\varepsilon^2} - \frac{e}{3}\frac{y^\lambda y^\rho}{y_\varepsilon^2}\left(\partial_\lambda F_{\rho\mu} + g_{\mu\lambda}J_\rho\right) + \frac{e}{6}J_\mu \log\frac{-y_\varepsilon^2}{\Lambda^2}\right\} + \mathcal{O}(y)$$

The red terms are odd in y, so they cancel in Schwinger's prescription. However, there is a logarithmic divergence due to the blue term, unless $J_\mu = 0$. Even then, the violet term leads to a direction dependence of the limit.

Omitting the parallel transport in Schwinger's prescription, even more divergences survive. Consider, for example a static external potential $A^\mu(x) = \delta_0^\mu V(\underline{x})$ and $v = e^0$. From the leading term, one then obtains

$$\frac{U(x+te^0, x)}{t^3} - \frac{U(x, x+te^0)}{t^3} = \frac{ieV(\underline{x})}{t^2} - \frac{i(eV(\underline{x}))^3}{6} + \mathcal{O}(t).$$

The need to remove these unwanted terms explains the appearance of supplementary renormalization conditions, for example in the computation of higher order corrections to the Uehling potential [Wichmann & Kroll 56].

An example in $1+1$ dimensions

Consider massless fermions in $1+1$ dimensions on a spatial interval of length L in the presence of a constant electric field E. Use bag boundary conditions

$$i\gamma^1\psi(\pm L/2) = \pm\psi(\pm L/2).$$

With $E_n = -(n+\frac{1}{2})\pi/L$, for $n \in \mathbb{Z}$, we have the normalized modes

$$\psi_n(x) = \frac{1}{\sqrt{2L}} \begin{pmatrix} \exp(\frac{i}{2}eE(\frac{L^2}{4} - x^2) - iE_n x) \\ -(-1)^n \exp(-\frac{i}{2}eE(\frac{L^2}{4} - x^2) + iE_n x) \end{pmatrix}.$$

In particular,

$$\bar{\psi}_n(x)\gamma^0\psi_n(x) = 1/L$$

is constant, so that the formal mode sum yields a vanishing (or constant) charge density, as claimed in the literature [Ambjørn & Wolfram 83].

The two-point function

$$\omega(\bar{\psi}(t,x)\psi(t',x')) = \sum_{n=0}^{\infty} \bar{\psi}_n(x)\psi_n(x')e^{-iE_n(t-t')}$$

can be explicitly computed. With parametric point-splitting, one obtains

$$\omega(j^0(x)) = \frac{1}{\pi}e^2 E x^1, \qquad \omega(T_{\mu\nu}(x)) = -\left(\frac{\pi}{24L^2} - \frac{e^2E^2(x^1)^2}{2\pi}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The Coulomb potential in 3 + 1 dimensions

Our aim is to compute the vacuum polarization $\rho(r)$ due to electrons in a Coulomb potential. We define the two-point function by filling up the negative continuum:

$$\omega(\bar{\psi}(x)\psi(x')) = \int_{(-\infty,-m^2]} \sum_{l,m} \bar{\psi}_{l,m}(E,\underline{x})\psi_{l,m}(E,\underline{x}')e^{iE(t-t')}\mathrm{d}E.$$

Consider the limit $x \to x'$ for $\underline{x} = \underline{x}'$:

$$\omega(\bar{\psi}(t,\underline{x})\gamma^0\psi(t',\underline{x})) = \int_{(-\infty,-m^2]} f(E,r)e^{iE(t-t')}\mathrm{d}E.$$

For the contraction of the parametrix, on the other hand, one obtains

$$\operatorname{tr}\gamma^0 H((t,\underline{x}),(t',\underline{x})) = \frac{2i}{\pi^2}\frac{U(t',t)}{(t'-t)^3} + \frac{im^2}{2\pi^2}\frac{U(t',t)}{t'-t} + \frac{1}{12\pi^2}J^0(\underline{x})\log\frac{t'-t}{\Lambda^2} + \mathcal{O}(t'-t).$$

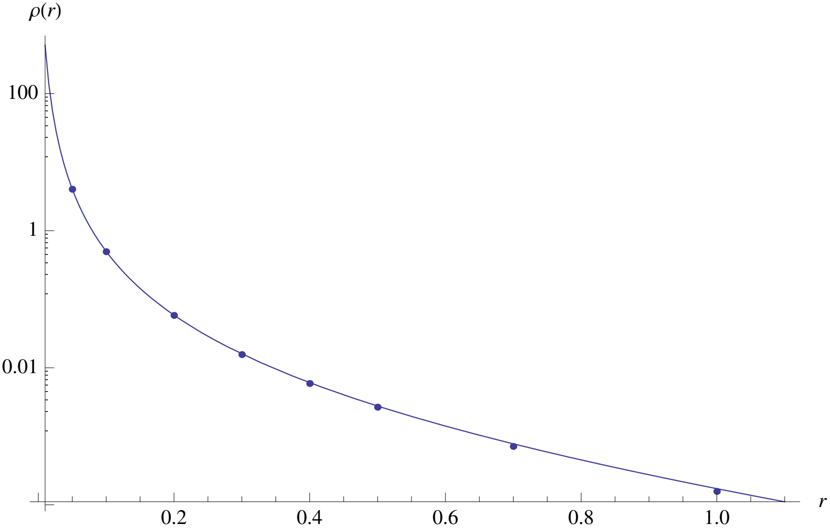
The red term vanishes away from the origin. The blue terms are as for the vacuum two-point function, up to the parallel transport $U(t',t)$, which can be accommodated as

$$e^{ieA_0(t-t')}\int_{(-\infty,-m^2]} f_0(E,r)e^{iE(t-t')}\mathrm{d}E = \int_{(-\infty,-m^2+eA_0(r)]} f_0(E-eA_0(r),r)e^{iE(t-t')}\mathrm{d}E.$$

Hence, the expectation value of the renormalized charge density can be written as

$$\langle\rho(r)\rangle = \int \left(\chi_{(-\infty,-m^2]}(E)f(E,r) - \chi_{(-\infty,-m^2+eA_0(r)]}(E)f_0(E-eA_0(r),r)\right)e^{iE(t-t')}\mathrm{d}E.$$

A comparison with the Uehling potential for small charge ($\alpha Z = 0.1$).



Time-dependent homogeneous field

For a time dependent homogeneous field $A_\mu = \delta_{3\mu} A(t)$, one can use the same trick for the computation of $\langle j^3(t) \rangle$ as for the computation of $\langle \rho(r) \rangle$ in the static case.

For the time-dependent field $A(t) = -E \tanh(wt)/w$, the normalized modes and the corresponding pair production probabilities are known explicitly [Narozhnyi & Nikishov 70].

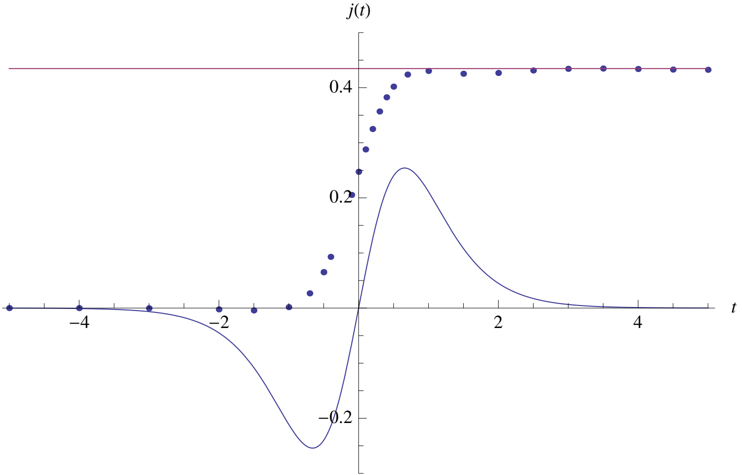
For the vacuum current $\langle j^\mu \rangle$ at linear order in the external potential, a corrected version of Serber's expression is:

$$\langle j^\mu(x) \rangle = \int \mathrm{d}^4x' \; K(x-x') \Box\Box J^\mu(x'),$$

$$K(x) = \frac{\alpha}{8\pi^2 \sqrt{x^2}} \int_0^{\pi/2} \mathrm{d}\psi \; \cos^4 \psi \int_m^\infty \mathrm{d}k \; J_1(2k\sqrt{x^2}/\cos\psi)/k^2.$$

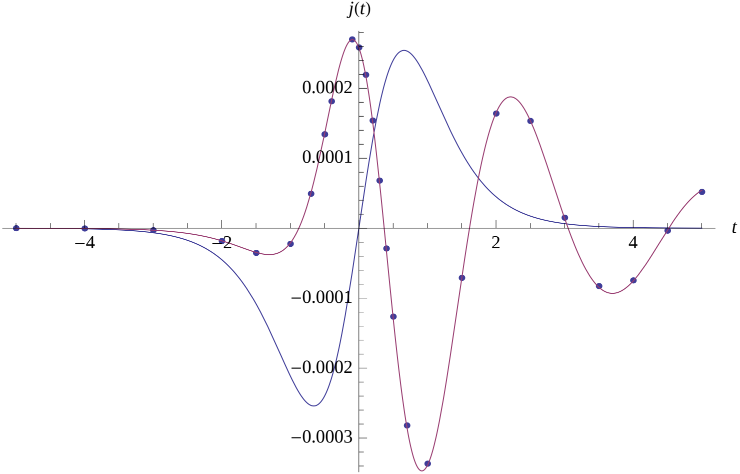
Super-critical peak field strength

For a super-critical peak field strength $E = 10E_c$, one sees how the asymptotic current is built up ($w = 1$ in units of m). The blue line shows the external current divided by 100, the violet line the asymptotic current, computed via the particle production probability.



Sub-critical peak field strength

For a sub-critical peak field strength $E = 0.1 E_c$, the asymptotic current is very small, but there are local oscillations ($w = 1$ in units of m). The blue line shows the external current divided by 1000 and the violet line the expression at $\mathcal{O}(E)$, due to Serber.



Outline

- Extension of the framework of locally covariant field theory to charged fields, such that gauge backgrounds and gauge transformations are treated on equal footing with gravitational backgrounds and isometries.
- Elementary calculation of anomalies.
- Possibly also relevant to QED in external potentials.

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