Categorial subsystem independence as morphism co-possibility

Miklós Rédei

Based on joint work with Z Gyenis

London School of Economics

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Preamble

von Neumann on his 1932 book on Mathematical Foundations of QM:

> *The subject-matter is partly physical-mathematical, partly, however, a very involved conceptual critique of the logical foundations of various disciplines (theory of probability, thermodynamics, classical mechanics, classical statistical mechanics, quantum mechanics). This philosophical-epistemological discussion has to be continuously tied in and quite critically synchronised with the parallel mathematical-physical discussion. It is, by the way, one of the essential justifications of the book, which gives it a content not covered in other treatises, written by physicists or by mathematicians, on quantum mechanics.*
>
> *(von Neumann to Cirker, October 3, 1949)*

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- We can perhaps risk conclusion:
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- More boldly:
  - mathematical physics is the technically explicit philosophy of physics
- OK, more carefully:
  - mathematical physics is the technically explicit philosophy of physics that is meaningful and useful

Main message

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- The standard notions of subystem independence in local quantum physics can be recovered as subobject independence by choosing the category of C^*-algebras with the injective C^*-algebra homomorphisms and the class of operations as morphism classes

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- Categorial subobject independence as morphism co-possibility is a natural independence concept that can be formulated in a general category
- The standard notions of subystem independence in local quantum physics can be recovered as subobject independence by choosing the category of C^*-algebras with the injective C^*-algebra homomorphisms and the class of operations as morphism classes
- Categorial subsystem indepedence is a natural axiom to express relativistic locality in the categorial approach to local quantum phyics

Outline

1. Categorial formulation of quantum field theory (Brunetti-Fredenhagen-Verch)
2. Subobject independence as morphism co-possibility in a general category
3. Categorial subsystem independence as subobject independence with respect to operations as morphisms
4. Why categorial subsystem independence is an attractive axiom

(Brunetti, Fredenhagen and Verch 2003)

Quantum field theory ... is a covariant functor ... in the ... fundamental and physical sense of implementing the principles of locality and general covariance...

R. Brunetti, K. Fredenhagen, R. Verch: "The generally covariant locality principle. A new paradigm for local quantum field theory"

Communications in Mathematical Physics **237** *(2003) 61-78*

Categorial formulation of quantum field theory

Main Idea (Fredenhagen, Brunetti and Verch 2003)

Formulate locality and general covariance in terms of the two categories:

- $(\mathfrak{Man}, hom_{\mathfrak{Man}})$
  category of spacetimes
  with isometric causal embeddings of spacetimes as morphisms
- $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$
  category of C^*-algebras
  with injective C^*-algebra homomorphisms as morphisms

The category $(\mathfrak{Man}, hom_{\mathfrak{Man}})$

- Objects $Obj(\mathfrak{Man})$: 4D C^∞ spacetimes (M, g) with Lorentzian metric g
  - time oriented, globally hyperbolic
- Morphisms $hom_{\mathfrak{Man}}$:

$$\psi \colon (M_1, g_1) \to (M_2, g_2)$$

  isometric smooth embeddings such that
  - ψ preserves time orientation
  - ψ is causal:
    if the endpoints $\gamma(a), \gamma(b)$ of a timelike curve
    $$\gamma \colon [a, b] \to M_2$$
    are in the image $\psi(M_1)$, then the whole curve is in the image:
    $$\gamma(t) \in \psi(M_1) \text{ for all } t \in [a, b]$$
  - composition of morphisms: usual composition of maps

The category $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$

- Objects $Obj(\mathfrak{Alg})$: C^*-algebras (unital)
- Morphisms: injective unit preserving C^*-algebra homomorphisms

$$\alpha\colon \mathcal{A}_1 \to \mathcal{A}_2$$

  - composition of morphisms: usual composition of C^*-algebra homomorphisms

Definition

A locally covariant quantum field theory is a covariant functor $\mathcal{F}$ between the categories $(\mathfrak{Man}, hom_{\mathfrak{Man}})$ and $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$

$$\begin{tikzcd}
(M, g) \arrow[r, "\psi"] \arrow[d, "\mathcal{F}"'] & (M', g') \arrow[d, "\mathcal{F}"] \\
\mathcal{F}(M, g) \arrow[r, "\mathcal{F}(\psi)"] & \mathcal{F}(M', g')
\end{tikzcd}$$

$$\begin{aligned}
\mathcal{F}(\psi_1 \circ \psi_2) &= \mathcal{F}(\psi_1) \circ \mathcal{F}(\psi_2) \\
\mathcal{F}(id_{\mathfrak{Man}}) &= id_{\mathfrak{Alg}}
\end{aligned}$$

- Covariance of $\mathcal{F}$ expressing general physical covariance
- How to express physical locality?

Physical locality

Physical locality $=$ **Causal Locality**

$\|$

The observational-operational properties of the physical systems localized in spacetime regions are in harmony with the causal relations between the spacetime regions.

Spacetime has a causal structure that specifies

- causally independent
  and spacetime regions
- causally dependent

$\Downarrow$

Causal locality conditions to be imposed on the functor $\mathcal{F}$ should regulate the behavior of $\mathcal{F}$ from the perspective of both causally independent and dependent spacetime regions.

Minimal expression of physical locality

The covariant functor of categorial quantum field theory

$$\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$$

should satisfy

- **Causal Locality – Independence**:
  - Einstein Causality
- **Causal Locality – Dependence**:
  - Time slice axiom

Call this axiom system BASIC

Locally covariant categorial quantum field theory – Einstein Causality

Definition

The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is called

- (Einstein) Causal if

$$\Big[\mathcal{F}(\psi_1)\Big(\mathcal{F}(M_1, g_1)\Big), \mathcal{F}(\psi_2)\Big(\mathcal{F}(M_2, g_2)\Big)\Big]_{-}^{\mathcal{F}(M,g)} = \{0\}$$

  whenever

$$\begin{aligned} \psi_1 &: (M_1, g_1) \to (M, g) \\ \psi_2 &: (M_2, g_2) \to (M, g) \end{aligned}$$

  and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M

Locally covariant categorial quantum field theory – Time slice axiom

Definition

If (M, g) and (M', g') and

$$\psi \colon (M, g) \to (M', g')$$

are such that $\psi(M, g)$ contains a Cauchy surface for (M', g') then

$$\mathcal{F}(\psi)\mathcal{F}(M, g) = \mathcal{F}(M', g')$$

In what sense is Einstein Causality (**not**) a causal independence condition?

Einstein Causality does entail:

- No superluminal signaling with respect to measurements represented by
  - non-selective, spatio-temporaly local projection postulate
  - non-selective spatio-temporaly local Kraus operations

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- Operational subsystem independence (Redei & Summers 2010): Any two (non-selective) operations performed on spacelike separated subsystems S_1, S_2 of system S are jointly implementable as a single operation on S

Categorial subsystem independence needed

Einstein Locality $\not\Rightarrow$ subsystem independence

$\Downarrow$

To formulate a Causal Locality– Independence condition expressing subsystem independence on the functor $\mathcal{F}$ representing a locally covariant quantum field theory we need a

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Can be done by interpreting

- subsystem $\Leftrightarrow$ subobject
- independence $\Leftrightarrow$ morphism co-possibility (Redei 2014)

Subobject in a category $(\mathfrak{C}, Hom_{\mathfrak{C}})$

Definition (of subobject)

A subobject of object C in $\mathfrak{C}$ is an equivalence class of monomorphisms

$$f_A \colon A \to C$$

where f_A is defined to be equivalent to monomorphism

$$f_B \colon B \to C$$

if there is an isomorphism

$$g \colon A \to B$$

such that

$$\begin{aligned} f_B \circ g &= f_A \\ f_B &= f_A \circ g^{-1} \end{aligned}$$

Independence of monomorphisms f_1, f_2

Definition (of independence of a pair of monomorphisms)

Monomorphisms

$$\begin{aligned} f_1 \colon C_1 &\to C \\ f_1 \colon C_2 &\to C \end{aligned}$$

are called $Hom_{\mathfrak{C}}$-independent in object C if for any two morphisms

$$\begin{aligned} m_1 \colon C_1 &\to C_1 \\ m_2 \colon C_2 &\to C_2 \end{aligned}$$

in $Hom_{\mathfrak{C}}$, there exists morphism $m \colon C \to C$ in $Hom_{\mathfrak{C}}$ such that

$$\begin{tikzcd} C_1 \arrow[r, "f_1"] \arrow[d, "m_1"'] & C \arrow[d, "m"] & C_2 \arrow[l, "f_2"'] \arrow[d, "m_2"] \\ C_1 \arrow[r, "f_1"] & C & C_2 \arrow[l, "f_2"'] \end{tikzcd}$$

Subobject independence

Definition

Two subobjects of object C represented by the two equivalence classes

$$|f_1| \text{ and } |f_2|$$

of monomorphisms f_1, f_2 into object C are called $Hom_{\mathfrak{C}}$-independent if any two monomorphisms

$$g_1 \text{ in the equivalence class } |f_1|$$
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Roughly: Two subobjects of object C are $Hom_{\mathfrak{C}}$-independent iff any two morphisms on any representations of the subobjects are jointly implementable by a single morphism on C

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  - *Hom*-independence of Boolean subalgebras A, B of C does not entail logical independence of A, B
  - Logical independence of Boolean subalgebras A, B of C does entail *Hom*-independence of A, B in C if A, B generate C

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  - Logical independence of Boolean subalgebras A, B of C does entail *Hom*-independence of A, B in C if A, B generate C
- von Neumann lattices (quantum logic) ?

Refining subobject independence

Observation: The class of morphisms defining the subobject relation need not be the same as the class of morphisms with respect to which subobject independence is defined

Given

- a class of morphisms $Mor_{\mathfrak{C}}$
- another class $Hom_{\mathfrak{C}}$ with respect to which the notion of subobject is defined

the notion of
$Mor_{\mathfrak{C}}$-independence of $Hom_{\mathfrak{C}}$-subobjects
is meaningful

as long as morphisms from the two classes can be composed

Example of refined subobject independence

Example

- One can consider subobjects in the category of C^*-algebras $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ with respect to the class of morphisms $hom_{\mathfrak{Alg}}$ containing injective unit preserving C^*-algebra homomorphisms
- One can take the class of operations $Op_{\mathfrak{ALG}}$ as the class of morphisms that is used to define subobject independence

$$\Downarrow$$

the notion of

$Op_{\mathfrak{ALG}}$-independence of $hom_{\mathfrak{Alg}}$-subobjects

is meaningful

Categorial subsystem independence

One can recover the major subsystem independence concepts that occur local quantum physics by choosing special subclasses of the class of all non-selective operations $Op_{\mathfrak{Alg}}$:

- States as a subclass of operations $\rightarrow$ C^*-independence
- Normal states as the subclass of operations $\rightarrow$ W^*-independence
- Normal operations as subclass of operations $\rightarrow$ operational W^*-independence
- Product versions of these specific independence concepts obtained by considering $Op_{\mathfrak{Alg}}$-independence in the product sense with respect to the respective subclasses of operations
  - C^*-and W^*-independence in the product sense
  - operational C^*-and W^*-independence in the product sense

$Op_{\mathfrak{Alg}}$-independence serves as a general, categorial frame in which subsystem independence can be formulated and analyzed

Causal Locality-Independence in QFT as subobject independence in categorial QFT

Definition

The covariant functor of categorial quantum field theory

$$\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$$

is said to satisfy the

$Op_{\mathfrak{Alg}}$-Causal Independence condition

if whenever

$$\psi_1\colon (M_1, g_1) \to (M, g)$$
$$\psi_2\colon (M_2, g_2) \to (M, g)$$

and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M

then the $hom_{\mathfrak{Alg}}$-subobjects $\mathcal{F}(\psi_1)(\mathcal{F}(M_1))$ and $\mathcal{F}(\psi_2)(\mathcal{F}(M_2))$ of object $\mathcal{F}(M)$ are $Op_{\mathfrak{Alg}}$-independent in $\mathcal{F}(M)$

Comments on $Op_{\mathfrak{Alg}}$-independence

- Interpretation of $Op_{\mathfrak{Alg}}$-independence:
  - Any two operations (e.g. measurements) on two subsystems located in spacelike separated spacetime regions of a larger system can be realized as a single operation on the larger system
  - subsystem independence condition
- $Op^{product}_{\mathfrak{Alg}}$-independence:
  - The operation m implementing the operations m_1, m_2 on the subalgebras factorizes over the subalgebras
  - Just a particular form of $Op^{product}_{\mathfrak{Alg}}$-independence — no new independence content

Requiring $Op_{\mathfrak{Alg}}$-Causal Independence as axiom

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If $Op_{\mathfrak{Alg}}$ is replaced by $Op_{\mathfrak{Alg}}^{product}$ then

Call this axiom system $\text{OPIND}^{\times}$

Replacing Einstein Locality by tensor property

One can (Brunetti & Fredenhagen 2009)

- extend the category $(\mathfrak{Man}, hom_{\mathfrak{Man}})$ to a tensor category $(\mathfrak{Man}^{\otimes}, hom^{\otimes}_{\mathfrak{Man}})$
- take the tensor category $(\mathfrak{Alg}^{\otimes}, hom^{\otimes}_{\mathfrak{Alg}})$ of C^*-algebras with respect to the minimal C^*-tensor product

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require the covariant functor of categorial quantum field theory

$$\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$$

to satisfy

- **Causal Locality – Independence**:
  - $\mathcal{F}$ is extendible to a tensor functor $\mathcal{F}^{\otimes}$ between the tensor categories $(\mathfrak{Man}^{\otimes}, hom^{\otimes}_{\mathfrak{Man}})$ and $(\mathfrak{Alg}^{\otimes}, hom^{\otimes}_{\mathfrak{Alg}})$
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Call this axiom sytem TENSOR

Split axiom

The covariant functor of categorial quantum field theory

$$\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$$

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- **Causal Locality – Independence**:
  - Einstein Locality
  - Categorial split property
  - Weak additivity
- **Causal Locality – Dependence**:
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Call this axiom sytem BASIC+SPLIT

Relation of axiom systems

$$
\begin{array}{ccc}
\textbf{BASIC+SPLIT} & \Leftrightarrow & \textbf{TENSOR} \\
 & & \Uparrow \quad \color{red}{\Downarrow ?} \\
 & & \textbf{OPIND}^{\times} \\
 & & \color{blue}{\Uparrow ?} \quad \Downarrow \\
 & & \textbf{OPIND} \\
 & & \Downarrow \not\Uparrow \\
 & & \textbf{BASIC}
\end{array}
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${\color{red}\Downarrow ?}$ Problem: nonextendability of operations in general
But ${\color{red}\Downarrow}$ holds if local algebras are injective

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- $\color{red}{\Downarrow ?}$ Problem: nonextendability of operations in general
  But $\color{red}{\Downarrow}$ holds if local algebras are injective
- $\color{blue}{\Uparrow ?}$ No strict proof in terms of models of axioms but:
  likely $\color{blue}{\not\Uparrow}$ because operational C^*-independence in the product sense is strictly stronger than operational C^*-independence

Tentative conclusion

- BASIC is too weak to express the full content of subsystem independence
- OPIND$^\times$/TENSOR/BASIC+SPLIT seems too strong

  contains more than required by subsystem independence
- OPIND has the right conceptual strength

  having direct interpretation: subsystem independence with respect to operations

Summary

- Relativistic locality as causal independence of physical systems localized in causally independent spacetime regions can be expressed in categorial quantum field theory in different ways
- There is a natural notion of categorial subobject independence as morphism co-possibility that could/should be investigated further
- Subsystem independence in local quantum physics can be formulated as a categorial subobject independence with respect to the operations on C^*-algebras as morphisms
- Categorial subsystem independence seems to be a natural axiom in categorial local quantum physics to express relativistic locality

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Categorial split property

Definition

The functor $\mathcal{F}$ has the split property if the following two conditions hold:

1. For spacetimes M, N and morphism $\psi \colon M \to N$ such that the closure of $\psi(M)$ is compact, connected and in the interior of M, there exists a type I von Neumann factor $\mathcal{R}$ such that

$$\mathcal{F}(\psi)(\mathcal{F}(M)) \subset \mathcal{R} \subset \mathcal{F}(N)$$

2. A continuity property of the $\mathcal{F}(\psi')$ with respect to the inclusion $\mathcal{R} \subset \mathcal{R}'$, where $\psi' : M \to L$ and

$$(\mathcal{F}(\psi') \circ \mathcal{F}(\psi))(\mathcal{F}(M)) \subset \mathcal{F}(\psi')(\mathcal{R}) \subset \mathcal{F}(\psi')(\mathcal{F}(N)) \subset \mathcal{R}' \subset \mathcal{F}(L)$$

Weak additivity

Definition

The functor $\mathcal{F}$ satisfies weak additivity if for any spacetime M and any familiy of spacetimes M_i with morphisms $\psi_i \colon M_i \to M$ such that

$$M \subseteq \cup_i \psi_i(M_i)$$

we have

$$\mathcal{F}(M) = \overline{\cup_i \mathcal{F}(\psi_i)(\mathcal{F}(M_i))}^{norm}$$

Motivation for categorial quantum field theory (Brunetti, Fredenhagen and Verch 2003)

Quantum field theory incorporates two main principles into quantum physics, locality and covariance. Locality expresses the idea that quantum processes can be localized in space and time (and, at the level observable quantities, that causally separated processes are exempt from any uncertainty relations restricting their commensurability). The principle of covariance within special relativity states that there are no preferred Lorentzian coordinates for the description of physical processes, and thereby the concept of an absolute space as an arena for physical phenomena is abandoned. Yet it is meaningful to speak of events in terms of spacetime points as entities of a given, fixed spacetime background, in the setting of special relativistic physics.

Categorial relativistic quantum field theory

In general relativity, however, spacetime points loose this a priori meaning. The principle of general covariance forces one to regard spacetime points simultaneously as members of several, locally diffeomorphic spacetimes. It is rather the relations between distinguished events that have physical interpretation.
This principle should also be observed when quantum field theory in presence of gravitational fields is discussed.

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