On Localization Of Infinite Spin Particles

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Based on a joint work with R.Longo and K.-H. Rehren:
"Where Infinite Spin Particles Are Localizable", arXiv:1505.01759.

What is a particle?

- The classical notion of particle as pointlike object is meaningless in a quantum theory (Heisenberg uncertainty relation).
- In Relativistic Quantum Mechanics, **particles are associated to positive energy unitary representations of the Poincaré group** (Wigner 1939). Representations should yield the states spaces of the simplest physical system - particles.

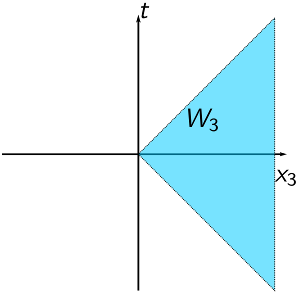
What are localized states of U?

- The language of standard subspace nets is useful to describe localization properties of one particle states.
- Brunetti, Guido and Longo in 2002 give a natural and canonical way to localize particles - modular localization

Wedge regions

A **wedge region** is a Poincaré transformed of W_3:

$$W_3 = \{p \in \mathbb{R}^{1+3} : |p_0| < p_3\},$$

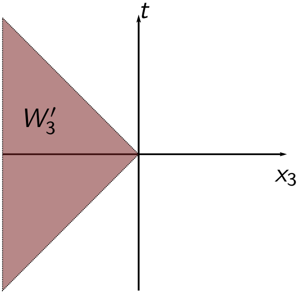


The set of wedge regions will be denoted by $\mathcal{W}$.

Wedge regions

The **causal complement** of W_3 is:

$$W_3' = \{p \in \mathbb{R}^{1+3} : |p_0| < -p_3\}$$



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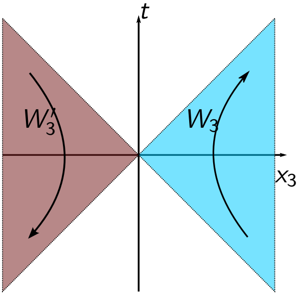
Wedge regions

To W_3 corresponds a pure Lorentz transformation, the **boost** fixing W_3:

$$\Lambda_3(t)(p_0, p_1, p_2, p_3) = (\cosh(t)p_0 + \sinh(t)p_3, p_1, p_2, \sinh(t)p_0 + \cosh(t)p_3)$$



Λ_W is the boost associated to $W \in \mathcal{W}$.

Modular Localization

Starting point is the **Bisognano and Wichmann property**: pure Lorentz transformation implemented by modular groups of standard subspaces associated to wedge subregions of the Minkowskii spacetime.
It always hold in **Wightman fields**.

U positive energy (anti-)unitary representation of $\mathcal{P}_+$ $\overset{\text{B-W}}{\rightarrow}$ canonical net of standard subspaces $\mathcal{W} \ni W \mapsto H(W) \subset \mathcal{H}$ with B-W on wedges

It is possible to define the subspace associated to a region $X \subset \mathbb{R}^{1+3}$ as

$$H(X) \doteq \bigcap_{W : W \supset X} H(W).$$

Second quantization of such nets give free fields. The construction is coordinate free (Wightman fields).

Modular localization and infinite spin particles

There are three families of particles (unitary Poincaré rep's). Infinite spin particles are usually considered unphysical.
Main results:

- It is **no possible** to associate a **Wightman field** to such infinite spin particles (Yngvason 1969).
- **Modular localization**: it is possible to define the canonical net of standard subspaces on wedges (and its second quantization) for infinite spin particles. It can be restricted to spacelike cone, but on double-cones the questions was still open. (BGL 2002)
- Infinite spin free fields are generated by fields localized on **semi-infinite strings - spacelike cone** (Mund, Schroer, Yngvason 2005)

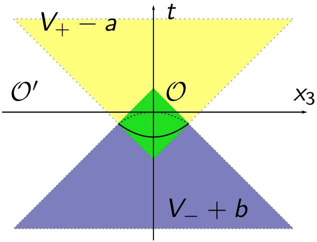
Question: are infinite spin particles localizable in some bounded regions?

Double cone

A **double cone region** is a causally closed region obtained as intersecting translations of a forward and a backward light cones:

$$\mathcal{O} = (V_+ + a) \cap (V_- + b)$$

where $V_+ = \{p \in \mathbb{R}^{1+3} : p^2 > 0, p_0 > 0\}$ and $V_- = -V_+$



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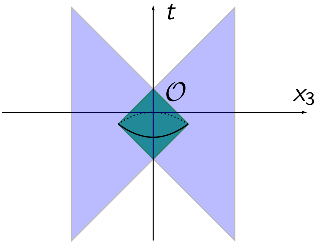

It can be equivalently obtained as intersection of wedges.

Outline

1. Preliminaries (one particle structure)
2. Main Result: Where Infinite Spin Particles are localizable
3. Generalization and counterexample

Part 1: one particle structure

Standard Subspaces

(Araki, Brunetti, Eckmann, Guido, Longo, Osterwalder, Rieffel, van Daele, ...)

Definition

We recall that a real linear closed subspace of an Hilbert space $H \subset \mathcal{H}$ is called **standard** if it is *cyclic* $(\overline{H + iH} = \mathcal{H})$ and *separating* $(H \cap iH = \{0\})$.

Symplectic complement of H:

$H' \equiv \{\xi \in \mathcal{H} : \Im\langle \xi, \eta \rangle = 0, \forall \eta \in H\} = (iH)^{\perp_{\mathbb{R}}}$

It can be stated the analogue of Tomita theory of standard subspace.

$$\left\{ \begin{array}{c} \text{Standard} \\ \text{subspace} \\ H \subset \mathcal{H} \end{array} \right\} \overset{1:1}{\longleftrightarrow} \left\{ \begin{array}{c} (J, \Delta)\text{anti-unitary} \\ \text{and self-adjoint} \\ \text{operators on } \mathcal{H} \text{ s.t.} \\ J\Delta J = \Delta^{-1} \end{array} \right\} \overset{1:1}{\longleftrightarrow} \left\{ \begin{array}{c} \text{closed, densely def.} \\ \text{anti-linear inv.} \\ S = J\Delta^{1/2} \end{array} \right\}$$

Remark

Let $A \subset B(\mathcal{H})$ be v.N.a. with a cyclic and separating vector Ω and $H = \overline{A_{sa}\Omega}$. Then the Tomita operators $S_{A,\Omega} = S_H$ coincide.

Unitary representations of the Poincaré group
Wigner 1939

- The Poincaré group is the group of the Minkowski spacetime isometries. First, we will consider its connected component of the identity $\mathcal{P}_+^\uparrow = \mathbb{R}^4 \rtimes \mathcal{L}_+^\uparrow$ on the 1+3 dimensional spacetime.

- **Irreducible unitary representations** of (the double covering of) the Poincaré group $\tilde{\mathcal{P}}_+^\uparrow = \mathbb{R}^4 \rtimes \mathrm{SL}(2,\mathbb{C})$ are all obtained induction.

- Fixed a point $q \in \mathbb{R}^{1+3}$ in the joint spectrum of translations, one induces from unitary representations of the **stabilizer subgroup**, namely $\overline{\mathrm{Stab}}_q$, of $\tilde{\mathcal{P}}_+^\uparrow$ w.r.t. q: $U = Ind_{\mathrm{Stab}_q \uparrow \tilde{\mathcal{P}}_+^\uparrow} V$.
  Actually it is enough to start with a representation of the **little group** $\mathrm{Stab}_q = \overline{\mathrm{Stab}}_q \cap \mathrm{SL}(2,\mathbb{C})$

- **Positivity of the energy**: translations joint spectrum in $\overline{V}_+$

Unitary representations of the Poincaré group

Wigner 1939

Massive representations:

- Choosing $q = (m, 0, 0, 0)$ the little group is SU(2)
- $U_{m,s}$ representations of mass $m > 0$ and spin $s \in \frac{\mathbb{N}}{2}$.

Massless representation:

- Choosing $q = (1, 0, 0, 1)$, the little group is the double cover of $E(2)$
- $\tilde{E}(2) = \mathbb{R}^2 \rtimes \widetilde{\mathrm{SO}}(2)$ representations are obtained by induction again. Starting the induction with a **positive or zero radius** in (the dual of) $\mathbb{R}^2$, we obtain two families of unit. rep's:
  - $V_{\kappa,\epsilon}$ $\kappa > 0$, $\epsilon = \{0, \frac{1}{2}\}$ if $V_{\kappa,\epsilon}$ **is faithful** (continuous family)
  - V_h, $h \in \frac{\mathbb{N}}{2}$ if **translation rep. is trivial** (discrete family)
- $U_{0,\kappa,\epsilon} = Ind_{\tilde{E}(2)\uparrow\widetilde{\mathcal{P}_+^\uparrow}} V_{\rho,\epsilon}$ Infinite Spin

  $U_{0,h} = Ind_{\tilde{E}(2)\uparrow\widetilde{\mathcal{P}_+^\uparrow}} V_n$ Finite Helicity

Standard subspaces Poincaré covariant nets

A Poincaré covariant "net" of standard subspace is a map $\mathcal{W} \ni W \longmapsto H(W) \subset \mathcal{H}$ associating to any wedge region W, a real linear subspaces of a Hilbert subspace of $\mathcal{H}$ s.t.

1. **Isotony:** if $W_1, W_2 \in \mathcal{W}$, $W_1 \subset W_2$ then $H(W_1) \subset H(W_2)$
2. **Poincaré Covariance and Positivity of the energy:** $\exists$ U positive energy representation of the proper orthochronous Poincaré group $\tilde{\mathcal{P}}_+^\uparrow$.
$$U(g)H(W) = H(gW), \qquad \forall W \in \mathcal{W}, \forall g \in \tilde{\mathcal{P}}_+^\uparrow.$$
3. **Reeh - Schlieder:** $\forall W \in \mathcal{W}$, $H(W)$ is a cyclic subspace of $\mathcal{H}$.
4. **Bisognano-Wichmann:** for every wedge $W \in \mathcal{W}$
$$\Delta_{H(W)}^{it} = U(\Lambda_W(-2\pi t))$$
5. **Wedge twisted locality:** For every wedge $W \in \mathcal{W}$, we have
$$ZH(W') \subset H(W)', \qquad with\, Z = \frac{1 + i\Gamma}{1 + i}$$
where $\Gamma = U(2\pi)$

Part 2: Where Infinite Spin Particles are localizable

Infinite spin representations have no dilations

Lemma

Let G be a locally compact group, $H \subset G$ a closed subgroup and β an automorphism of G such that $\beta(H) = H$.
If V is a unitary representation of H and $U \equiv Ind_{H\uparrow G} V$, then

$$U \cdot \beta \simeq Ind_{H\uparrow G} V \cdot \beta^0, \qquad \text{where } \beta^0 \equiv \beta|_H.$$

Let δ_t be the dilation automorphism of $\tilde{\mathcal{P}}_+^\uparrow$ s.t.

$$\delta_t(g) = g\,, \forall g \in \mathcal{L}_+^\uparrow, \quad \delta_t(p) = e^t p,\ p \in \mathbb{R}^4.$$

If U was dilation covariant $U_\kappa \cdot \delta_t \simeq U_\kappa$.

Let α_t be the $\tilde{\mathcal{P}}_+^\uparrow$ the automorphism implemented by boosts in 3-direction $\alpha_t(q = (1,0,0,1)) = (e^t, 0, 0, e^t)$.
$U_\kappa \cdot \alpha_t \simeq U_\kappa$ as α is inner.

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We can define the $\tilde{\mathcal{P}}_+^\uparrow$ automorphism

$$\beta_t = \alpha_{-t} \cdot \delta_t$$

and $\beta_t(q) = q$, $\beta(\tilde{\mathcal{P}}_+^\uparrow) = \tilde{\mathcal{P}}_+^\uparrow$ ($\Rightarrow \beta_t(\mathrm{Stab}_q) = \mathrm{Stab}_q$).
Clearly $U_\kappa \cdot \beta_t \simeq U_\kappa \cdot \delta_t$.

Proposition

Let $U_\kappa \simeq Ind_{\overline{\mathrm{Stab}_q}\uparrow\tilde{\mathcal{P}}_+^\uparrow} \bar{V}_\kappa$ be an infinite spin, irreducible unitary representation of $\tilde{\mathcal{P}}_+^\uparrow$. Then

$$U_\kappa \cdot \beta_t \simeq U_{\kappa'}$$

where $\kappa' = e^{-t}\kappa$.

Corollary

Let U be an irreducible, positive energy, unitary representation of $\tilde{\mathcal{P}}_+^\uparrow$.
Then U is dilation covariant iff U is massless with finite spin.

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Double cone localization implies dilation covariance

A consequence of the Huygens theorem for massless K-G equation:

Lemma

Assume that U is a massless, unitary representation of $\tilde{\mathcal{P}}_+^\uparrow$ acting covariantly on a twisted-local net of standard subspaces on double cones. Let O_1, O_2 be double cones with O_2 in the timelike complement of O_1, then

$$H(O_2) \subset ZH(O_1)' \ .$$

Consequence: $H(V_+) \doteq \overline{\bigvee_{O\subset V_+} H(O)}$ is standard!

Proposition

Let U be a massless representation of $\tilde{\mathcal{P}}_+^\uparrow$, acting covariantly on a net H of standard subspaces on wedges satisfying properties 1–5.
If $H(O)$ is cyclic for some double cone O, then U is dilation covariant as $D(2\pi t) = \Delta_{H(V_+)}^{-it}, t \in \mathbb{R}$.

It follows by Borchers' Theorem and Bisognano Wichmann property.

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Infinite spin particles are not localizable on double cone

Our main result:

Theorem

Let U be an irreducible unitary, infinite helicity, representation of $\tilde{\mathcal{P}}_+^\uparrow$ and

$$\mathcal{W} \ni W \longmapsto H(W) \subset \mathcal{H}$$

be a U-covariant net of standard subspaces, satisfying 1-5.

Then

$$H(\mathcal{O}) \doteq \bigcap_{W \supset \mathcal{O}} H(W) = \{0\}$$

for any double cone $\mathcal{O}$.

First and Second quantization

"The first quantization is a mystery, the second is a functor" (E.Nelson)

- **First quantization net**: $\mathbb{R}^{1+s} \supset \mathcal{O} \longmapsto H(\mathcal{O}) \subset \mathcal{H}$ satisfying 1-5.
- **Second quantization net**: a map

$$\mathbb{R}^{1+s} \supset \mathcal{O} \longmapsto R_{\pm}(\mathcal{O}) \subset \mathcal{F}_{\pm}(\mathcal{H})$$

where $\mathcal{F}_{\pm}$ is the symmetric (resp. anti-symmetric) Fock space.

$$R_{+}(H) \equiv \{w(\xi) : \xi \in H\}'' , \quad R_{-}(H) \equiv \{\Psi(\xi) : \xi \in H\}'' ,$$

with $w(\xi)$ the Weyl unit's and $\Psi(\xi)$ the Fermi field op's on $\mathcal{F}_{\pm}(\mathcal{H})$.
We have a (free) net of local algebras satisfying relativistic and quantum basic assumptions.

Theorem

Let $R_{\pm}$ be the free Bose/Fermi infinite spin free field net.
Then $R_{\pm}(C)$ is cyclic on the vacuum vector if C is a spacelike cone, but $R_{\pm}(O) = \mathbb{C} \cdot 1$ if O is any bounded spacetime region.

Remark: needed abstract duality $R_{+}(H)' = R_{+}(H')$, $R_{-}(H) = ZR_{-}(iH')Z^{*}$ to prove that $R_{\pm}(\cap_a H_a) = \bigcap_a R_{\pm}(H_a)$ (Leyland, Roberts, Testard; Foit).

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The interacting case

In an interacting theory:

Theorem

Let U be a positive energy unitary representation of $\mathcal{P}_+^\uparrow$ acting covariantly on a double cone localized, isotonous, local, net of von Neumann algebras

$$\mathcal{W} \ni \mathcal{O} \longmapsto \mathcal{A}(\mathcal{O}) \subset \mathrm{B}(\mathcal{H}),$$

with a unique vacuum vector $\Omega \in \mathcal{H}$, s.t.

- Bisognano and Wichmann property hold
- Reeh-Schlieder property holds for double cones

Then U has no infinite spin in its direct integral disintegration (up to a null measure set).

The proof relies on the fact that the associated standard subspace net

$$\mathcal{W} \ni W \longmapsto H(W) = \overline{\mathcal{A}(O)_{sa}\Omega}$$

decomposes accordingly to the U direct integral disintegration (by B-W) and we conclude using the result for standard subspaces (by R-S).

Part 3: Generalization and counterexample

$s+1$ dimensional case ($s \geq 2$)

- Infinite spin representations in $\mathbb{R}^{s+1}$ are massless representation induced by (unitary) faithful representation of the little group.
- They are a continuous family: fixing $q = (1, 1, 0, \ldots, 0) \in \mathbb{R}^{s+1}$

  $$\mathrm{Stab}_q = \tilde{E}(s-1) \subset \tilde{\mathcal{P}}_+^\uparrow$$

  where $\tilde{\mathcal{P}}_+^\uparrow$ is the universal covering of $\mathcal{P}_+^\uparrow$ in $s+1$ spacetime dimensions and $\tilde{E}(s-1)$ is the double cover of the $s-1$ dimensional Euclidean group (we just consider bose/fermi alternative in 2 space dimensions.)
- In even space dimensions Huygens principle does not hold but one can show that spacelike (twisted) locality implies

  $$H(O_1) \subset iZH(O_2), \qquad O_1 \subset O_2^t.$$

In these cases any net of standard subspaces satisfying 1-5, undergoing an infinite spin representation have trivial double cones subspaces.

Bisognano and Wichmann is an essential assumption:

Let V be a real, bosonic, unitary representation of $SL(2,\mathbb{C})$ on an Hilbert space $\mathcal{K}$: there exists a real vector space $K \subset \mathcal{K}$ s.t. $K + iK = \mathcal{K}$, $JK = K$, $V(\mathrm{SL}(2,\mathbb{C}))K = K$.

Let U_0 be the scalar, zero mass, unitary irreducible representation of $\tilde{\mathcal{P}}_+$. Let

$$\mathcal{W} \ni W \mapsto H(W) \in \mathcal{H}$$

the canonical BGL-net associated to U_0.

We can define the **new standard subspaces** net

$$\tilde{H} : \mathcal{W} \ni W \longmapsto K \otimes H(W) \subset \tilde{\mathcal{H}} \doteq \mathcal{K} \otimes \mathcal{H}$$

Counter-example
B-W property is necessary

There are **two representations** acting on $\tilde{H}$:

$$U_I : \widetilde{\mathcal{P}_+^\uparrow} \ni (a, A) \longmapsto 1_{\mathcal{K}} \otimes U_0(a, A) \in \mathcal{U}(\tilde{\mathcal{H}})$$

$$U_V : \widetilde{\mathcal{P}_+^\uparrow} \ni (a, A) \longmapsto V(A) \otimes U_0(a, A) \in \mathcal{U}(\tilde{\mathcal{H}})$$

Double cones subspaces

$$\tilde{H}(\mathcal{O}) \doteq \bigcap_{W \supset \mathcal{O}} \tilde{H}(W) = K \cap (\cap_{W \supset \mathcal{O}} H(W))$$

are cyclic and separating.

- If V does not contain the trivial representation then U_V is purely infinite spin.
- Bisognano and Wichmann property holds for U_I (not for U_V).

Thanks for your attention