Some scale-invariant states of quantum spin chains and their properties.

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Vanderbilt
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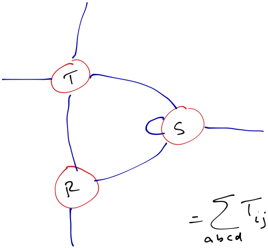
April 5, 2017

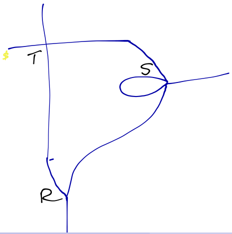
Tensor networks



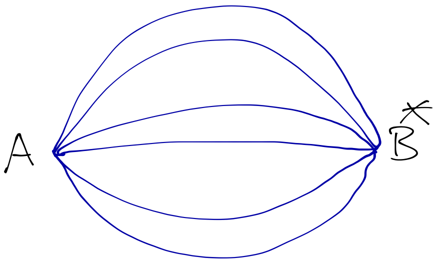
$$= X_{ijkl}$$

$$= \sum_{abcd} T_{ijad} S_{akcbb} R_{dcl}$$

or better



NB: The inner product $< A, B >$ of two tensors is given by:



$$= \sum_{i_1 \dots i_j} A_{i_1 i_2 \dots i_j} \overline{B_{i_1 i_2 \dots i_j}}$$

Quantum spin chains

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where $h : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$ is a selfadjoint operator giving the interaction between two adjacent spins.

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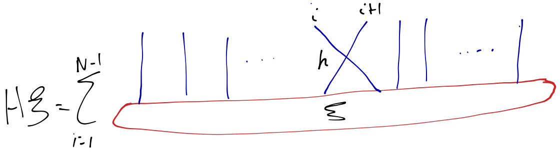

Transfer Matrix

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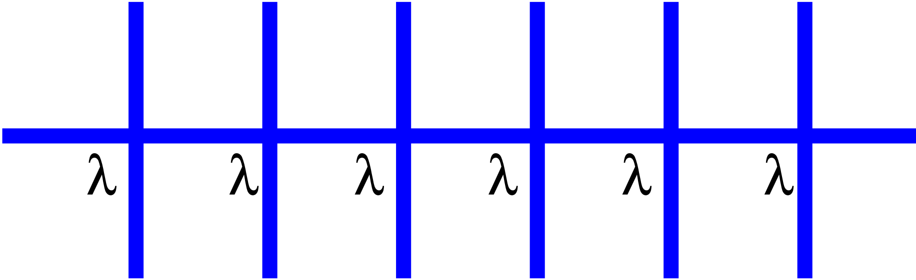
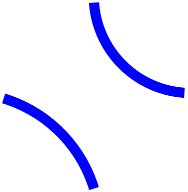
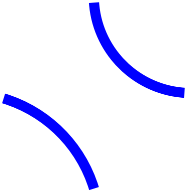


(Where we have represented $t(\lambda)$ by just λ.)

Further suppose that $t(0) =$ 

Further suppose that $t(0) =$ 

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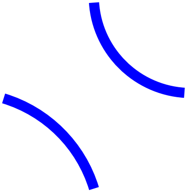
Further suppose that $t(0) =$ 

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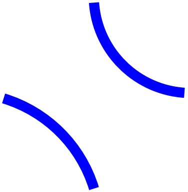
Further suppose that $t(0) =$ 

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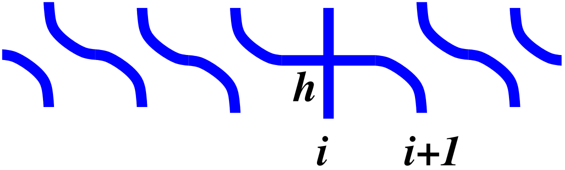
$$T'(0) = \sum_i$$



where $h = t'(0)$.

So

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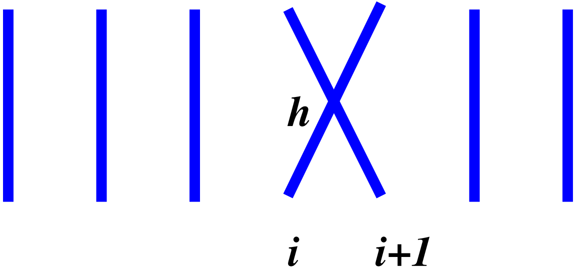

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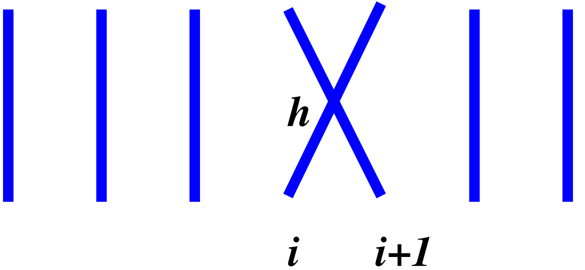
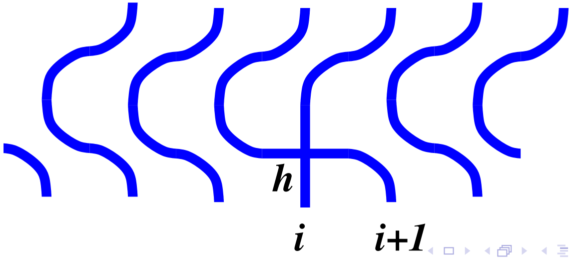


and in case I went too quickly here's the intermediate step:



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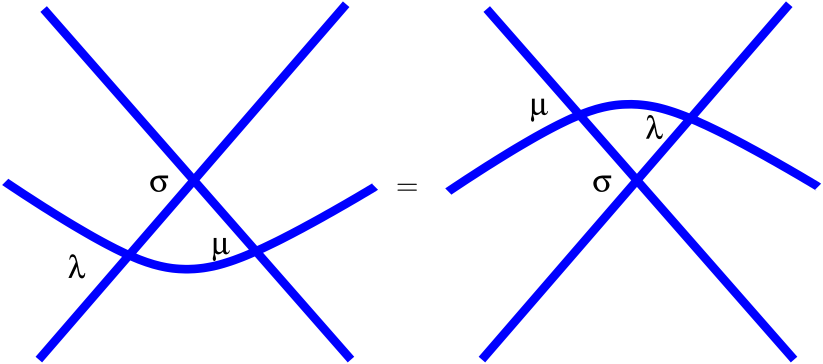

CTM

By a cute argument YBE implies

$$T(\lambda)T(\mu) = T(\mu)T(\lambda)$$

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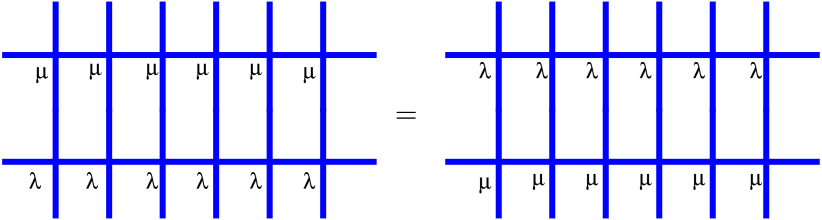

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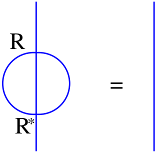
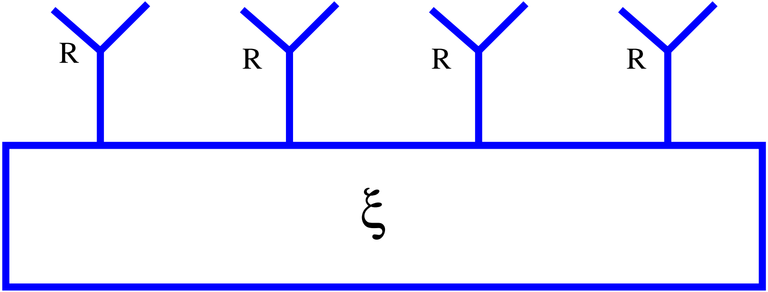
(The adjoint R^* would be the spin blocking operator.)
The isometry condition is :



Equipped with this R we may now construct an increasing family of Hilbert spaces $\mathfrak{H}_n$ of dimension $(dim\mathcal{H})^{2^n}$ by embedding $\otimes^{2^n}\mathcal{H}$ in $\otimes^{2^{n+1}}\mathcal{H}$ via the following tensor network (planar tangle):



If we choose a unit vector Ω in $\mathcal{H}$ it defines a vector Ω in each $\otimes^{2^n}\mathcal{H}$ via the above embedding. We will call it the vacuum vector.

Definition

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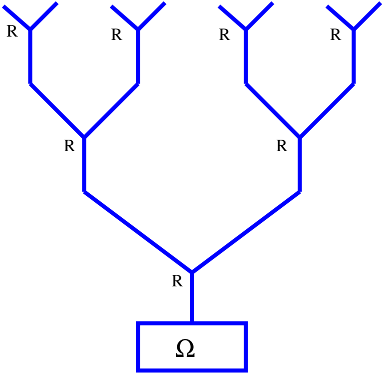

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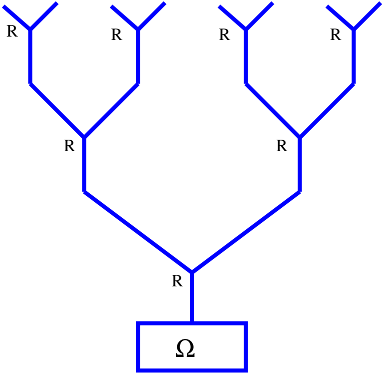

From now on we will tend to suppress R

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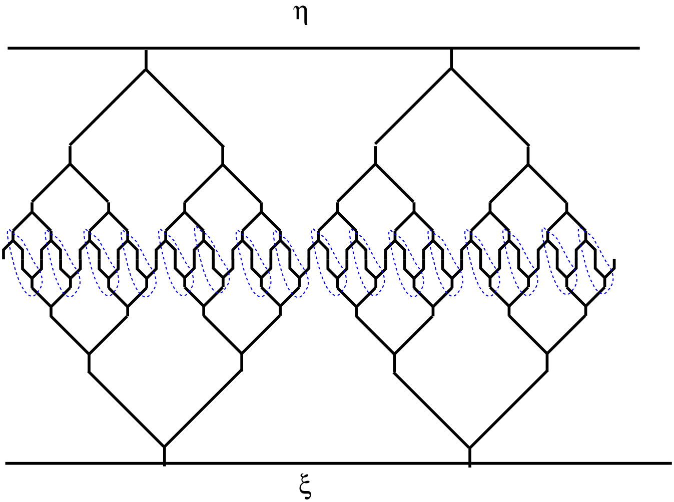
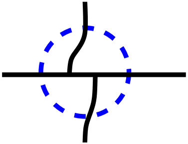
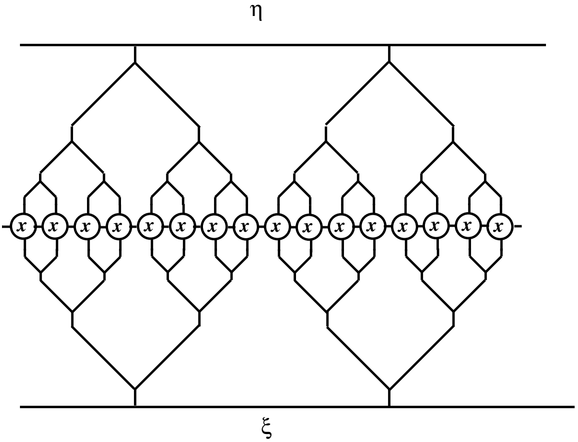


η
ξ

Now all the regions in the blue dotted circles can be isotoped to look like



so if we call x the element inside the box with 4 legs, the picture becomes:



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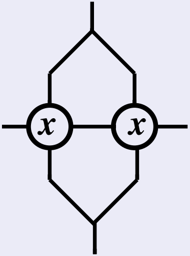
We define the quadratic renormalisation map $\mathcal{R}(x)$ by



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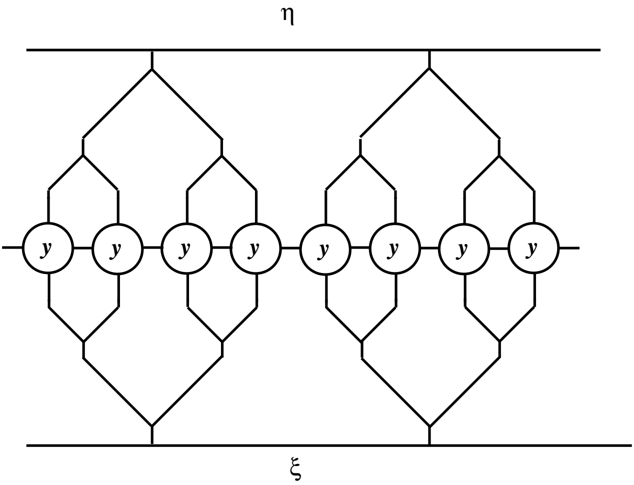

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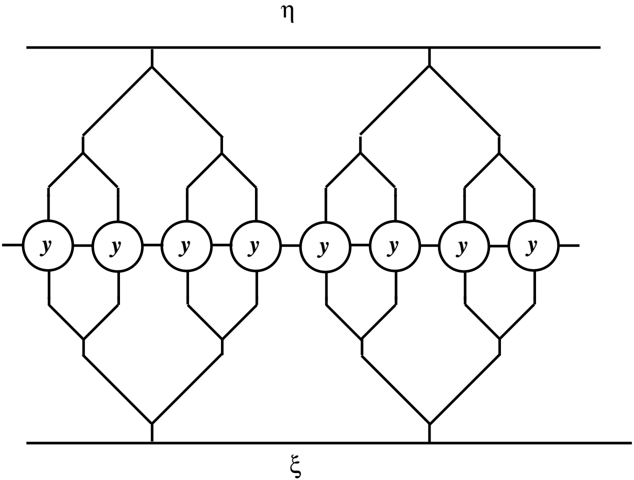

Continuing in this way we see that

$$\langle \rho_{\frac{1}{2^{k+n+1}}} \xi, \eta \rangle = \langle T_{2^k}(\mathcal{R}^n(x))\xi, \eta \rangle$$

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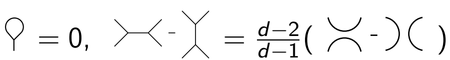
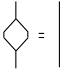
 , and of course unitarity, .

With these relations it is not hard to show that:

$$\mathcal{R}(a) = \{\frac{d^2-5d+7}{(d-1)^2}p^2 + 2pq + 2\frac{d-2}{d-1}pr + q^2 + r^2\}$$

$$-$$

$$\{\frac{1}{(d-1)^3}p^2 + \frac{1}{d-1}(2pq+q^2)\}$$

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Note that d in the above is the quantum dimension which can be $4cos^2\pi/n - 1$ for $n = 6, 7, 8, \cdots$ and $d = 3$ is the case of $SO(3)$-invariant tensors.

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In either case it is a very structured white noise as we may renormalise by the rate at which $< \rho_{\frac{1}{2^n}}(\xi), \eta >$ tends to zero to obtain *Two quadratic forms on the semicontinous limit* to which the renormalised $< \rho_{\frac{1}{2^n}}(\xi), \eta >$ converge.

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Indeed, define

$$[\xi, \eta]_n = \frac{< \rho_{\frac{1}{2^n}}(\xi), \eta >}{< \rho_{\frac{1}{2^n}}(\Omega), \Omega >}$$

Theorem

There are two quadratic forms on $\mathfrak{H}_R$, $Q_\pm$ such that

$$lim_{n\to\infty}[\xi, \eta]_{2n} = Q_+(\xi, \eta)$$

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These two quadratic forms should no doubt be called topsy turvy momenta....

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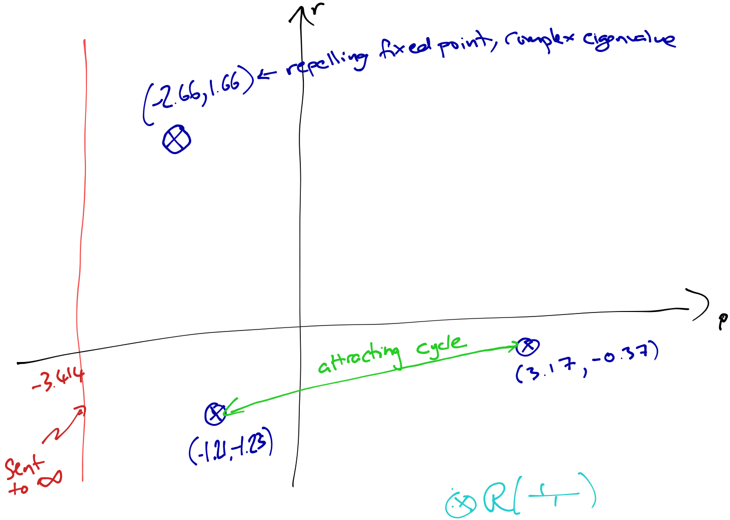
Here is a picture of the interesting points in $\mathbb{R}^2$ for the projectivised renormalisation.


r
(-2.66, 1.66) ← repelling fixed point, complex eigenvalue
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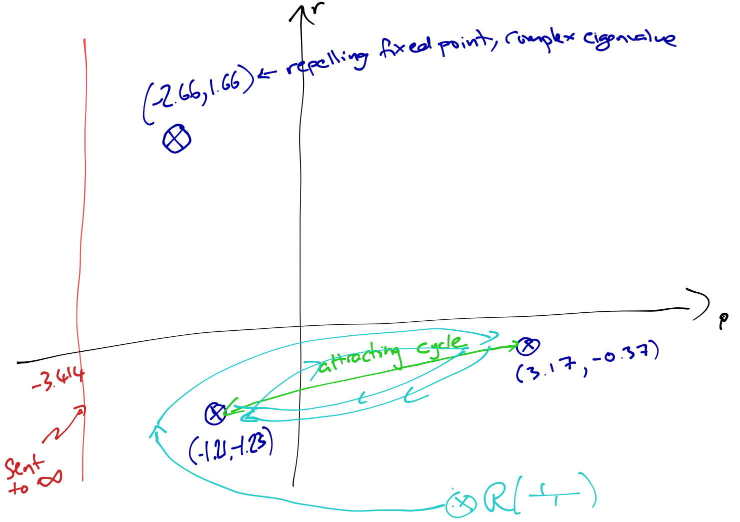
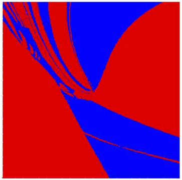


Scale invariant fractal behaviour can be observed by dividing the plane according to which of these two a point converges.



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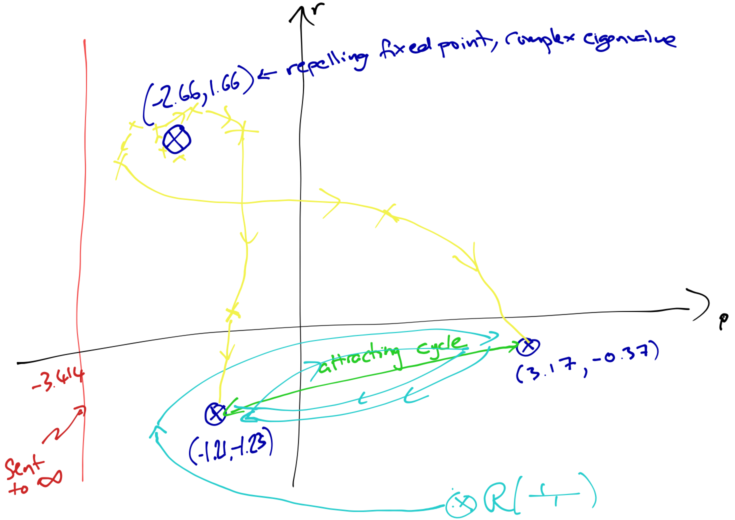
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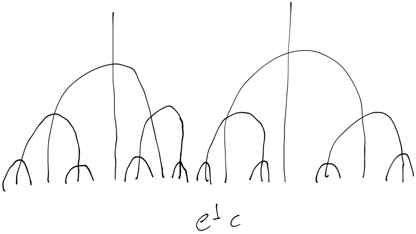

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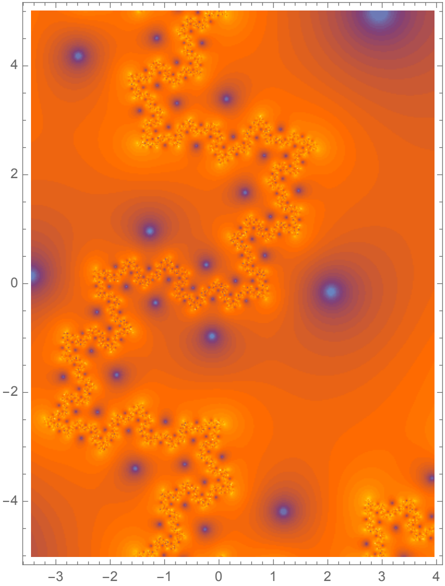
And here is a picture of its Julia set.



4
2
0
−2
−4
−3
−2
−1
0
1
2
3
4