A Hilbert space setting for higher spin interactions which replaces gauge theory

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Intention

view talk as confirmation of Borcher's paper in millennium edition of JMP "On revolutionizing QFT with Tomita-Takesaki modular theory" [1]

Lit. J. Mund, B. Schroer and J. Yngvason [2]
B. Schroer, A Hilbert space setting.., [4]
B. Schroer, *Modular localization and..* ,to appear in SHPMP [5]
(also B. Schroer, *Dark matter and Wigner's* ..[6])
J. Mund and B. Schroer, J. Mund, to appear

Modular localization (intrinsic formulation of causal localization) essential to understand QFT content of Wigner's "infinite spin" repr. Result: "noncompact matter" generated by semi-infinite spacelike strings. Was "initial ignition" of Hilbert space setting for $s \geq 1$ interactions (which are nonrenormalizable in pointlike setting).

$$pointlike\ nonrenorm.\ is\ localization\ problem \rightarrow stringl.\ renorm.$$

generic tightest localization consistent with H-positivity. $\Psi(x, e)$ are Wightman fields

BRST gauge ctnd.

Clash between pointlike massive vectorpotentials and Hilbert space positivity, one way out: scrap Hilbert space
Intuitive idea: lower short distance scaling dimension $d_{s.d} = 2$ of Proca field by compensation with neg. metric free Stückelberg field

$$A^K(x) \simeq A^P(x) + \partial_\mu \phi^K \quad \textit{in Krein space}, \;\; d(A^K) = 1 = d(\phi^K)$$

To convert equivalence relation into operator relation in Krein space need extension by ghosts $u, \tilde{u}$, BRST

$$sA^K_\mu = \partial_\mu u, \;\; s\phi^K = u, \;\; s\tilde{u} = -(\partial A^K + m^2 \phi^K)$$
$$sB^K = [Q, B^K]_{\text{grad}}, \;\; Q \textit{ ghost charge}, \;\; Q^2 = 0 \curvearrowright s^2 = 0$$

Formalism is simple but return to physics remains difficult, $s(F(x) = 0 \curvearrowright F(x)$ is local observable is o.k., but what about its extension to nonlocal operators as $s(S) = 0 \curvearrowright S$ physical? The combinatorial pert. control is consistent, although its physical content (beyond local observables) remains questionable.

Vectormesons in Hilbert space

With an s=1 pointlike Proca field

$$A_\mu^P(x) = \frac{1}{(2\pi)^{3/2}} \int e^{ipx} \sum_{s_3} u_\mu(p,s_3) a^*(p,s_3) + h.c.$$

$$\left\langle A_\mu^P(x)\ A_\nu^P(x') \right\rangle = \frac{1}{(2\pi)^3} \int e^{-ip(x-x')} M_{\mu\nu}(p) \frac{d^3p}{2p_0}, \quad M_{\mu\nu}(p) = -g_{\mu\mu'} + \frac{p_\mu}{m}$$

one can connect two stringlocal fields (loc. on $x + \mathbb{R}_+ e$, $e^2 = -1$)

$$A_\mu(x,e) = \int_0^\infty F_{\mu\nu}(x+se)e^\nu ds,\ F_{\mu\nu}(x) = \partial_\mu A_\nu^P(x) - \partial_\nu A_\mu^P(x)$$

$$\phi(x,e) = \int_0^\infty A_\mu^P(x+se)e^\mu ds,\ stringlocal\ scalar,\ "escort"\ of\ A$$

$$A_\mu(x,e) = A_\mu^P(x) + \partial_\mu\phi(x,e),\ d_{sd}(A^P) = 2 \rightarrow 1$$

$$A_\mu(x,e),\ \phi(x,e)\ covariant,\ stringlocal,\ d = 1$$

A^P, A, ϕ members of local equivalence (Borchers) class.

continued

$$\langle \Phi_1(x,e)\Phi_2(x',e')\rangle = \frac{1}{(2\pi)^{3/2}}\int e^{-ip(x-x')}M_{\Phi_1,\Phi_2}(p;e,e')\frac{d^3p}{2p_0},$$

$$M_{A_\mu,A_\nu} = -g_{\mu\nu} + \frac{e\cdot e' p_\mu p_\nu}{(pe-i\varepsilon)(pe'+i\varepsilon)} + \frac{p_\mu e_\nu}{pe-i\varepsilon} + \frac{p_\nu e'_\mu}{pe'+i\varepsilon},\ similar\ M_{A,\phi},\ ..$$

Achievement: maintain $H-space$ by ceding on localization but maintain covariance; e is (like x) a fluctuating localization variable.

- **Thm** (Buchholz, Fredenhagen) local observables (vacuum sector)+mass gap $\longrightarrow$ higher charge sectors generated by operators localized in arbitrarily narrow spacelike cones (cores: semi-infinite strings). For $s \geq 1$ one needs them. Pointlike (integer) spin $s \geq 1$ always nonrenorm. ($d_{int} > 4$), can stringlocal counterparts be renormalizable ?

$$d_s^P = s+1,\quad d_s^{st} = 1\ independent\ of\ s$$

$$\curvearrowright\ many\ stringlocal\ renormalizable\ couplings\ with\ \ d_{int} \leq 4$$

examples

examples: massive QED

$$L^P(x) = j^\mu(x) A_\mu^P(x) = L(x,e) - \partial^\mu V_\mu(x,e),\ L = j^\mu A_\mu,\ V_\mu = j_\mu \phi$$
$$d(L^P) = 5,\ \ d(L) = 4,\ d(V_\mu) = 4$$

The the d=5 derivative term "peels off" the leading dimension, equality in adiabatic S-matrix limit $S^{(1)} = \int L^P d^4x = \int L d^4x$ or

$$d_e(L - \partial V) = 0,\ \textit{differential form calculus in } d = 1+2 \ \textit{ de Sitter}$$

continued

- a more **subtle example**: Coupling to Hermitian field (factor g omitted)

$$L^P = A^P \cdot A^P H + cH^3 = L - \partial V, \;\; d_e(L - \partial V) = 0 \curvearrowright$$

$$L = A \cdot AH + cH^3 + A \cdot \phi \overleftrightarrow{\partial} H - \frac{m_H^2}{2}\phi^2 H \;\; (+u\, \tilde{u} H)$$

$$V_\mu = A_\mu \phi H + \frac{1}{2}\phi^2 \overleftrightarrow{\partial_\mu} H$$

comment: in BRST $L \to L^K$ (all fields$\to$ $field^K$) in Krein space, $d_e \to s$ i.e. $s(L - \partial V) = 0$ and $(+u\, \tilde{u} H)$

Formulation is different from Uni Zürich group (Scharf..) but computations are similar.

- **important observation**: The pair L, V_μ can also be constructed from $d_e(L - \partial V) = 0$, "self-induction" : field content$+d_{int} \leq 4$ fixes L-V pair

continued

- **induction of first order Y-M**: start with most general renormalizable coupling between 3 massive vectormesons and their escort ϕ

$$\sum f^1_{abc} F^{a,\mu\nu} A^b_\mu A^c_\nu, \ \sum f^2_{abc} A^{a,\mu} A^b_\mu \phi^c, \ \sum f^3_{abc} A^{a,\mu} \partial_\mu \phi^b \phi^c, \ \sum f^4_{abc} \phi^a \phi^b \phi^c$$

then self-induction relates the large number of independent couplings to just one, the induced L, V pair turns out to be

$$L = \sum \varepsilon_{abc} (F^{a,\mu\nu} B^b_\mu A^c_\nu + m^2 B^{a,\mu} A^b_\mu \phi^c), \ V^\mu = \sum \varepsilon_{abc} F^{a,\nu\mu} (A^b_\mu + B^b_\nu) \phi^c$$

similar arguments in BRST (Scharf), but here without "gauge principle", just H-positivity

- **Conclusion**: Lie-algebraic structure result from locality in Hilbert space, no need to impose it in the form of a "gauge principle" in Krein space, $s = 1$, QFT interactions do not need classical "crutches" (fibre bundles).

second order anomalies

clash between pointlike localization and H-space positivity (solved by: pointl. gauge$\rightarrow$stringl.Hilbert

- The singularities of time-ordering prevent that $(L' := L(x', e'))$

$$d_e TLL' - \partial^\mu TQ_\mu L' = 0, \;\; Q_\mu = d_e V_\mu, \;\; BRST: \;\; d_e \rightarrow s_{left}$$

$$same \;\; with \;\; (x, e) \leftrightarrow (x', e')$$

$$\curvearrowright (d_e + d_{e'})(TLL' - \partial^\mu TV_\mu L' - \partial'^\nu TLV'_\nu + \partial^\mu \partial'^\nu TV_\mu V'_\nu) = 0$$

  formally from the first order relation; is a 2^{nd} order renormalization requirement. The $d = d_e + d_{e'}$ invariance corresponds to the BRST gauge invariance $d \simeq s$ (the nilpotent BRST s-operation). Last line is definition of pointlike $TL^P L^{P\prime}$

- Denoting by T_0 the naive time ordering

$$\left\langle T_0 \partial_\mu \varphi^*(x) \partial'_\nu \varphi(x') \right\rangle := \partial_\mu \partial'_\nu \left\langle T_0 \varphi^*(x) \varphi(x') \right\rangle, \;\; etc.$$

continued

These anomalies in the previous propagators lead to anomalies in the one-contraction components

$$A := d_e(T_0 LL' - \partial^\mu T_0 V_\mu L')_{1-contr.} = -d_e N + \partial^\mu N_\mu$$
$$\textit{from terms as } \ \partial^\mu \left\langle T_0 \partial_\mu \varphi^*(x)\varphi(x')\right\rangle = -i\delta(x-x') \ + reg.$$

and a similar formula for $A' = A(with\ x, e \longleftrightarrow x', e')$.

- The N and N_μ are renormalization terms for $T_0 LL'$ and $T_0 V_\mu L'$. Only the N contributes to the S-matrix. A nontrivial N, N_μ leads to finite renormalization of $TLL', TV_\mu L'$ i.e. $T_0 LL' \to T_0 LL' + \delta(x-x')L_2$
- The second order S-matrix does not depend on $\partial^\mu(T_0 V_\mu L' + N_\mu)$ derivative terms.

continued

- Massive spinor QED $N = N' = 0$, no induced L_2 contributions.
- Massive scalar QED, $L_2 = \varphi^* \varphi A \cdot A'$, as expected from gauge symmetry (set $e = e'$). L_2 term can be absorbed into $T_0 \to T$ [4]
- abelian H-model. 1^{st} order pair L, V_μ is more involved and so is 2^{nd} . Again $A \cdot AH^2$, $A \cdot A\phi^2$ terms absorbed into change of T-product, in the BRST formalism there remains [7]

$$TL^K L^{K\prime} - i\delta(x - x')R_{scharf}$$

$$R_{Scharf} = -\frac{m_H^2}{2m^2}(\phi^2 + H^2)^2, \ \ V_{Scharf} \equiv g^2 R + first\ order\ H, \phi - terms$$

$$V_{Scharf} = g^2 \frac{m_H^2}{8m^2}(H^2 + \phi^2 + \frac{2m}{g}\phi)^2 - \frac{m_H^2}{2} H^2, \ \ Mex.hat\ pot.$$

Higgs

Confirmed in new H-space setting ($s \rightarrow d_e$, $e = e'$ in S-matrix). There was no symmetry breaking and no physical reason to write it this way!! (with inverse g); V_{Scharf} is simply the combined sum of first order and second order terms, no g^{-1} terms.

- Apart from differences in the normalization of ϕ_K and the physical escort ϕ the S-matrix in the Hilbert space setting is the same (after taking $e = e'$). Strictly speaking the S-matrix calculation is outside the vacuum sector of the BRST gauge setting. But the H-space calculation offeres an posteriori support for such calculation. Quotation from Raymond Stora: "gauge theory is a miracle and I do not understand the origin of its consistency as far as we know it". Most physicists from the older generation (including myself) knew that quantum gauge theory is a successful "placeholder" for a still unkown perturbative renormalization theory for $s \geq 1$. Mandelstam and DeWitt attempted to go beyond gauge theory. Is the present H-space formulation in terms of stringlocal vectormesons the solution? Only additional work can tell.

continued

- The Mexican hat potential is a device to break gauge invariance of pointlike QED (nonsensical!) by shift in φ and in this way generate a mass

$$L = gj_\mu(x)A^\mu(x) + c(\varphi^*(x)\varphi(x))^2$$
$$j_\mu(x) := i\varphi^*(x)\overleftrightarrow{D}_\mu\varphi(x),\ D_\mu = \partial_\mu - iA_\mu(x)$$

- It also appears as a second order induced contribution in a BRST gauge *symmetry preserving* (sensical) AAH coupling of $m > 0$ A_μ
- The Hilbert space setting confirms correctness of induction mechanism: no breaking of gauge svmmetry of scalar QED but rather the implementation of the BRST gauge formalism for $A - H$ coupling.
- General structural thm.: charge of Maxwell-current of $m > 0$ vectormeson $j_\mu^{Max} = \partial^\nu F_{\mu\nu}$ is screened : $Q^{Max} = 0$. For complex matter two currents; Maxwell and (p-p̄) counting charge; coalesce for $m \rightarrow 0$. H-couplings: only Maxwell current.

LHC

- The H can be decoupled by Occam's razor $\rightarrow$ free massive H, (hardly fundamental). The misunderstanding of spontan. broken symm. starts when one identifies its physical meaning with a shift in field space. Intrinsic of s.s.b.

  $$\partial^\mu j_\mu = 0,\ Q = \int j_0 = \infty \overset{Thm}{\curvearrowright} m = 0\ \mathit{Goldst},\ \mathit{boson\ is\ culprit}$$

  The shift in field space is only a technical trick to generate a first order interation which leads to such a current. There is a big difference between "massaging a Lagrangian" and understanding the physics of what one is doing.

In couplings of massive vectormesons there is no s.s.b., Mex. hat potential results from second order *induction*.

- Possible foundational explanation of LHC exp.: scalar massive "gluonium" (possible interpolating fields ϕ), H is phenomenological description, similar to hadrons in QCD.

important lesson

- $s \geq 1$ *renormalizable interactions in Hilbert space* cannot be understood by "massaging" pointlike Lagrangian (the gauge-symmetry-breaking field shifts). Sometimes meaningless manipulations lead to apparently consistent results (the self-healing power of QFT) but to find the conceptual correct aspects is the task of the particle theorist.
- The $s \geq 1$ subtleties cannot be encoded into Feynman diagrams. It is now clear what experts always suspected: one needs a new renormalization theory for QFT (always in H-space) of higher spin interactions.
- It is easy to find stringlocal interactions with $d_{int} \leq 4$ and linear relations between $d = s + 1$ pointlike fields and their $d = 1$ stringlike siblings; example $s = 2$

$$g_{\mu\nu}(x,e) = g_{\mu\nu}(x) + \partial_\mu \phi_\nu|_{sym}\acute{} + \partial_\mu \partial_\nu \phi, \ \ \textit{two escorts}$$

but only $L's$ which belong to an $d_e(L - \partial V) = 0$ are physical. Very different from quantizing *E-H* actions.

continued

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