The Paradigm of Local Covariance

CJ Fewster
University of York

Hamburg, December 2017

- Genesis
- The paradigm
- Illustrations
- Coleman–Mandula & SPASs

Influences and inspirations from Klaus Fredenhagen's work

- Microlocal analysis
- Local covariance
- Fields in AQFT [Fredenhagen–Hertel]
- Scaling limits and universal type
- Split property
- SJ states [Brum–Fredenhagen]
- Universal algebra
- ...and probably more besides.

Commun. Math. Phys. 237, 31–68 (2003)
Digital Object Identifier (DOI) 10.1007/s00220-003-0815-7

Communications in **Mathematical Physics**

The Generally Covariant Locality Principle – A New Paradigm for Local Quantum Field Theory

Romeo Brunetti[1], Klaus Fredenhagen[1], Rainer Verch[2]

[1] II. Inst. f. Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany. E-mail: brunetti@na.infn.it; fredenha@x4u2.desy.de
[2] Inst. f. Theoretische Physik, Universität Göttingen, Bunsenstrasse 9, 37073 Göttingen, Germany. E-mail: verch@theorie.physik.uni-goettingen.de

Received: 23 January 2002 / Accepted: 5 September 2002
Published online: 2 April 2003 – © Springer-Verlag 2003

Dedicated to Rudolf Haag on the occasion of his eightieth birthday

Origins

Among others:

- Haag–Kastler (1964) 'net ideology'
  Brought into sharp relief by Fredenhagen's universal type result
- Wald's axioms for renormalizing $T_{\mu\nu}$ (SET) (1977)
- Kay's approach to the Casimir effect (1979)
  renormalize SET between the plates 'as if' in Minkowski

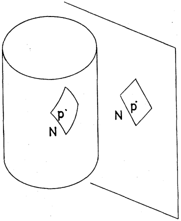

FIG. 1. The structure of field operators on the cylinder space-time is locally identical with that of ordinary flat space-time.

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  renormalize SET between the plates 'as if' in Minkowski
- Dimock's functorial discussion of global isometries (1980)
- Kay's F-Locality (1992, Haag's 70th birthday)
  on sufficiently small scales, local algebras of regions in (non)globally hyperbolic spacetimes should coincide with what would be obtained by regarding the local region as a spacetime in its own right

Crystallization

- Microlocal Analysis and Interacting Quantum Field Theories: Renormalization on Physical Backgrounds
  Brunetti–Fredenhagen 2000
- made possible by the microlocal revolution Radzikowski
- renormalization performed up to finitely many free functions

"The main open point in this paper is the fixing of the finite renormalizations. One expects that they can be chosen in terms of local functions of the metric, but a precise formulation meets a lot of problems. A similar problem was studied (and partially solved) in the definition of the expectation value of a renormalized energy momentum tensor of free fields by R. Wald [61]. We hope to return to this problem..."

Following intensive discussions at Oberwolfach (October 2000)

- Spin statistics connection Verch `math-ph/0102035`
- Local Wick polynomials... Hollands–Wald `gr-qc/0103074`
- BFV paper (Haag's 80th birthday) `math-ph/0112041`

Fundamental idea

Regard a physical theory as a mathematical object.

$$\text{Theory} : \text{Spacetimes} \to \text{Physical Systems}$$

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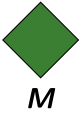

To each spacetime $\boldsymbol{M}$ there is a mathematical object $\mathcal{A}(\boldsymbol{M})$ describing physics on $\boldsymbol{M}$ according to theory $\mathcal{A}$.

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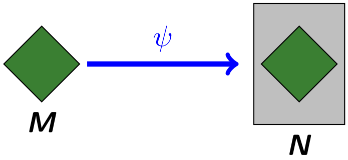

To each spacetime $\boldsymbol{M}$ there is a mathematical object $\mathcal{A}(\boldsymbol{M})$ describing physics on $\boldsymbol{M}$ according to theory $\mathcal{A}$.

To each suitable embedding of spacetimes $\psi : \boldsymbol{M} \to \boldsymbol{N}$, there is a suitable map $\mathcal{A}(\psi) : \mathcal{A}(\boldsymbol{M}) \to \mathcal{A}(\boldsymbol{N})$ embedding the physics on $\boldsymbol{M}$ as a subsystem of the physics on $\boldsymbol{N}$.

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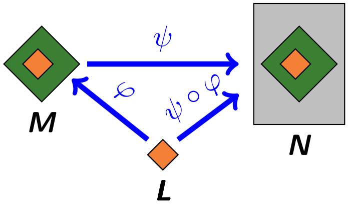

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Under successive embeddings: $\mathcal{A}(\varphi \circ \psi) = \mathcal{A}(\varphi) \circ \mathcal{A}(\psi)$.

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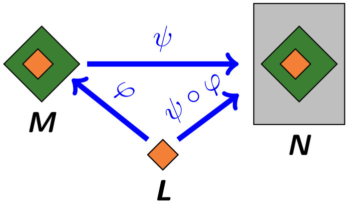

Local covariance: A theory is a functor from a category of (globally hyperbolic) background spacetimes to a category of physical systems

$$\mathcal{A} : \mathsf{BkGrnd} \to \mathsf{Phys}$$

Morphisms in Phys are monic – represent subsystem embeddings.

For QFT, Phys is a category of unital $*$-algebras.

We discuss BkGrnd later

Interpretation: constructive use of ignorance

- The causal complement of our spacetime region is unknowable
- Nonetheless, we can successfully conduct physics
- Conclusion: our ignorance cannot matter

Local covariance is a consistency mechanism guaranteeing that ignorance of spacetime beyond the region in question is irrelevant.

NB Topological data may have a different status.

Typical choices of BkGrnd

Typical choice for scalar fields is BkGrnd = Loc, defined with

Obj oriented & time-oriented globally hyperbolic spacetimes (of fixed dimension, finitely many components)

Mor hyperbolic embeddings: isometric embeddings preserving time and space orientations with causally convex image.

Typical choices of BkGrnd

Typical choice for scalar fields is BkGrnd = Loc, defined with

- **Obj** oriented & time-oriented globally hyperbolic spacetimes (of fixed dimension, finitely many components)
- **Mor** hyperbolic embeddings: isometric embeddings preserving time and space orientations with causally convex image.

Alternatives, allowing for general spin, include SpinLoc, FLoc

- **Obj** $\mathfrak{M} = (\mathcal{M}, e)$ where $\mathcal{M}$ is a smooth manifold of dimension n on which $e = (e^\nu)_{\nu=0}^{n-1}$ is a global coframe, from which a Loc object can be built.
- **Mor** $\psi : (\mathcal{M}, e) \to (\mathcal{M}', e')$ must induce a Loc-morphism and obey $\psi^* e' = e$.

Can add bundles, external fields, or use categories fibred over Loc

The dictionary

Physical concept	Mathematical realization
Theory	Functor $\mathcal{A}$
Subtheory	Natural transformation $\mathcal{A} \stackrel{.}{\to} \mathcal{B}$
Equivalence of theories	Natural isomorphism
Global gauge group	$\mathrm{Aut}(\mathcal{A})$
Einstein causality	Monoidal structure
Dynamics	Timeslice property
Action	Relative Cauchy evolution
Fields	'Naturals' $\Phi : \mathcal{D} \stackrel{.}{\to} \mathcal{A}$ forming an abstract $*$-algebra $\mathsf{Fld}(\mathcal{A})$
Additivity	Joins of subobjects
State space	Subfunctor of $\mathcal{A}^*_{+,1}$ (contravariant)

Mostly present already in BFV. Subtheories CJF + Verch gauge group CJF and monoidal structure Brunetti, Fredenhagen, Imani, Rejzner appeared later.

Dynamics: Time-slice axiom and relative Cauchy evolution

$\psi : \boldsymbol{M} \to \boldsymbol{N}$ is Cauchy if $\psi(\boldsymbol{M})$ contains a Cauchy surface of $\boldsymbol{N}$.
A locally covariant theory $\mathcal{A}$ satisfies the timeslice axiom if

$$\psi \text{ is Cauchy} \implies \mathcal{A}(\psi) \text{ is an isomorphism}$$

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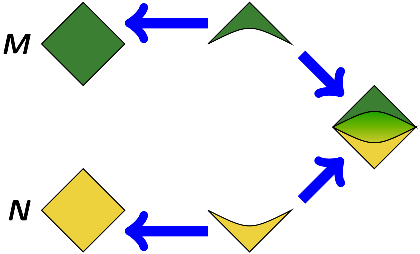
Rigidity of local covariance Any two spacetimes with oriented diffeomorphic Cauchy surfaces are linked by Cauchy morphisms.
Fulling, Narcowich & Wald 1981; CJF & Verch 2011



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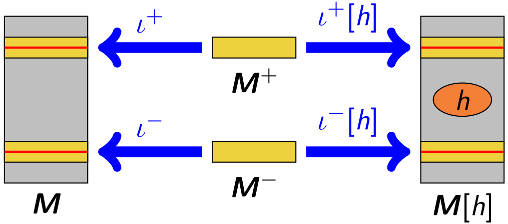

If $\mathcal{A}$ obeys timeslice, any metric perturbation h preserving global hyperbolicity defines an automorphism of $\mathcal{A}(\boldsymbol{M})$,

$$\mathrm{rce}_{\boldsymbol{M}}[h] = \mathcal{A}(\iota^-) \circ \mathcal{A}(\iota^-[h])^{-1} \circ \mathcal{A}(\iota^+[h]) \circ \mathcal{A}(\iota^+)^{-1}$$

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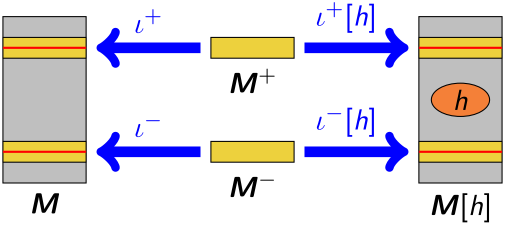

The functional derivative gives a stress-energy tensor:

$$[\mathrm{T}_{\boldsymbol{M}}(f), A] = -2i \left.\frac{d}{ds}\mathrm{rce}_{\boldsymbol{M}}[h(s)]A\right|_{s=0} \qquad f = \left.\frac{dh(s)}{ds}\right|_{s=0}.$$

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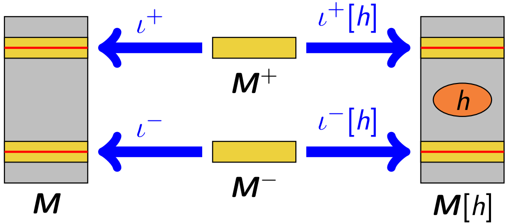

Variation of other background structures gives conjugate currents.
Zahn, CJF & Schenkel; applied to describe background independence by Brunetti, Fredenhagen, Rejzner

What use is local covariance?

- Renormalization
  - freedom reduced to constants, Hollands & Wald
  - renormalization of gauge theories Hollands, Fredenhagen & Rejzner
- Structural analysis of QFT
  - Transfer of results/properties from Minkowski by ridigity, e.g., Reeh–Schlieder, split, modular nuclearity, spin-statistics... Verch, CJF, Sanders, Lechner
  - Superselection theory Brunetti, Ruzzi
  - Non-existence of preferred states CJF & Verch
  - Coleman-Mandula analogue CJF
  - Higher structures Benini, Schenkel+
- Physical applications or influences
  - local thermal observables & Unruh Buchholz+... Verch
  - superselection for massless theories Buchholz & Roberts
  - cosmology Dappiaggi, Fredenhagen, Hack, Pinamonti, Verch, Degner
  - Casimir energy CJF & Pfenning, Marecki
  - quantum gravity Brunetti, Fredenhagen, Rejzner

Illustration I: Timeslice and Einstein causality combined

If $\boldsymbol{M}$ = Minkowski space and D_i are causally disjoint diamonds

$$\mathcal{A}(\boldsymbol{M}) \otimes \mathcal{A}(\boldsymbol{M}) \cong \mathcal{A}(D_1) \otimes \mathcal{A}(D_2) \cong \mathcal{A}(D_1 \sqcup D_2) \hookrightarrow \mathcal{A}(\boldsymbol{M})$$

(timeslice, plus Einstein causality).

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$$\mathcal{A}(\boldsymbol{M})^{\otimes k} \hookrightarrow \mathcal{A}(\boldsymbol{M}) \qquad \forall k \in \mathbb{N}$$

Every $\boldsymbol{N} \in \mathrm{Loc}$ has subspacetime $\boldsymbol{D}$ with Cauchy surface B^{n-1}, so

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Consequence: excluding trivial cases with $\dim \mathcal{A}(\boldsymbol{M}) \leq 1$,

$$\dim \mathcal{A}(\boldsymbol{N}) = \infty \qquad \forall \boldsymbol{N} \in \mathrm{Loc}$$

Note the geometrical nature of the argument.

Illustration II: Naturality, nuclearity and $\mathbb{N}$ CJF 2013

The preceding argument shows that it is problematic to interpret $\mathcal{A}(\boldsymbol{M})^{\otimes k}$ as k copies of the physics described by $\mathcal{A}(\boldsymbol{M})$.

Old wisdom: algebras are less important than their ‘relative position’, and only net morphisms are significant.

LCQFT replaces nets by functors, and net morphisms by naturals.

Regard $\mathcal{A}(\boldsymbol{M}) \hookrightarrow \mathcal{B}(\boldsymbol{M})$ as significant iff it is achieved ‘in the same way’ in all spacetimes, i.e., a sub-theory embedding

$$\begin{tikzcd} \mathcal{A}(\boldsymbol{M}) \arrow[r, "\zeta_{\boldsymbol{M}}"] \arrow[d, "\mathcal{A}(\psi)"] & \mathcal{B}(\boldsymbol{M}) \arrow[d, "\mathcal{B}(\psi)"] \\ \mathcal{A}(\boldsymbol{N}) \arrow[r, "\zeta_{\boldsymbol{N}}"] & \mathcal{B}(\boldsymbol{N}) \end{tikzcd}$$

Illustration II (ctd)

Any subtheory embedding $\mathcal{A}^{\otimes k} \dot{\to} \mathcal{A}$ induces an ouroboros endomorphism

$$\mathcal{A} \dot{\to} \mathcal{A}^{\otimes k} \dot{\to} \mathcal{A}$$

that is not an automorphism (unless $\mathcal{A} \cong \mathcal{A}^{\otimes k}$).

Example: Hilbert's hotel, $\mathcal{A}^{\otimes \aleph_0}$, with endomorphism

$$\eta_{\boldsymbol{M}} A_1 \otimes A_2 \otimes \cdots = \mathbf{1} \otimes A_1 \otimes A_2 \otimes \cdots .$$

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Given suitable nuclearity criteria, however, one may show: CJF, 2012

Theorem: All endomorphisms of a [suitable] locally covariant theory are automorphisms and the global gauge group is compact.

Nuclearity & naturality allow us to count.

Illustration III: Characterisation vs construction CJF & Verch

QFT in CST has tended to regard theories in terms of their constructions. LCQFT can provide characterisations.

Comparison: $\mathbb{R}$ can be

- constructed as the completion of $\mathbb{Q}$
- characterised as an ordered field with the l.u.b. property.

The characterisation is more practically useful than the construction.

Illustration III: Characterisation vs construction CJF & Verch

Example: let

- $P : \mathcal{D} \dot{\to} \mathcal{D}$ be the Klein-Gordon operator
- $E : \mathcal{D}^{\otimes 2} \dot{\to} \mathbb{C}$ be the advanced-minus-retarded bisolution

regarded as naturals. Then, a real Klein–Gordon theory is a pair $(\mathcal{A}, \Phi)$ where $\mathcal{A} : \mathsf{Loc} \to \mathsf{Alg}$, $\Phi \in \mathsf{Fld}(\mathcal{A})$, s.t.

$$\Phi^* = \Phi, \qquad \Phi \circ P = 0, \qquad \overbrace{[\Phi, \Phi] = iE1}^{\text{bilocal fields}}$$

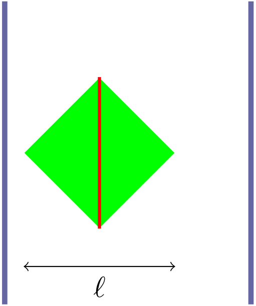
and with the universal property that for each such pair $(\mathcal{B}, \Psi)$, $\exists$ a unique $\alpha : \mathcal{A} \dot{\to} \mathcal{B}$ s.t.

$$\Psi = \alpha \cdot \Phi, \qquad \text{i.e.} \quad \Psi_{\boldsymbol{M}}(f) = \alpha_{\boldsymbol{M}} \Phi_{\boldsymbol{M}}(f)$$

$(\mathcal{A}, \Phi)$ is characterised uniquely up to equivalence in this way.

QFT in CST, without the spacetimes!

Illustration IV: Casimir effect CJF & Pfenning; Marecki



Cavity V of arbitrary geometry.

Apply Minkowski QEIs to an inertial line with $\boldsymbol{x} = \boldsymbol{x_0}$ within the diamond to find

$$\rho_{\mathcal{C}} \geq -\frac{C}{\ell^4}\,.$$

for the Casimir energy density $\rho_{\mathcal{C}}$. Optimise over ℓ to obtain an *a priori* bound

$$\rho_{\mathcal{C}} \geq -\frac{C}{\mathrm{dist}(\boldsymbol{x_0}, \partial V)^4}$$

based on locality, QEIs and simple geometry.

An analogue of the Coleman–Mandula Theorem CJF 2016

Although generic curved spacetimes have no symmetries, the Lorentz group acts functorially on FLoc.

Each $\Lambda \in \mathcal{L}_0$ corresponds to a functor $\mathcal{T}(\Lambda) : \mathsf{FLoc} \to \mathsf{FLoc}$

$$\mathcal{T}(\Lambda)(\mathcal{M}, e) = (\mathcal{M}, \Lambda e), \qquad (\Lambda e)^\mu = \Lambda^\mu{}_\nu e^\nu$$

and so that $\mathcal{T}(\Lambda)(\psi)$ has the same underlying map as ψ.

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Now define an action of $\mathcal{L}_0$ on theory $\mathcal{A} : \mathsf{FLoc} \to \mathsf{Phys}$ by

$${}^\Lambda\mathcal{A} = \mathcal{A} \circ \mathcal{T}(\Lambda)$$

i.e., $\mathcal{A}$ acting on 'pre-rotated frames'. NB: ${}^{\Lambda\Lambda'}\mathcal{A} = {}^{\Lambda'}({}^\Lambda\mathcal{A})$.

If the choice of frame is irrelevant, then $\mathcal{A}$ and ${}^\Lambda\mathcal{A}$ are equivalent for all Λ, and $\mathcal{A}$ is called $\mathcal{L}_0$-covariant.

$\mathcal{L}_0$-covariance requires the existence of natural isomorphisms

$$\eta(\Lambda) : \mathcal{A} \stackrel{.}{\to} {}^{\Lambda}\mathcal{A}, \qquad \eta(1) = \mathrm{id}_{\mathcal{A}}.$$

Comparing successive Lorentz transformations Λ_1, Λ_2 vs $\Lambda_2\Lambda_1$ gives a non-abelian group 2-cocycle (ξ, ϕ) of $\mathcal{L}_0$ valued in $\mathrm{Aut}(\mathcal{A})$.

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Theorem: $\mathcal{A}$ induces a canonical cohomology class

$$[\mathcal{A}] = [(\xi, \phi)] \in H^2(\mathcal{L}_0; \mathrm{Aut}(\mathcal{A}))$$

and therefore a group extension of $\mathcal{L}_0$ by $\mathrm{Aut}(\mathcal{A})$

$$1 \to \mathrm{Aut}(\mathcal{A}) \to E \to \mathcal{L}_0 \to 1$$

Locally covariant fields transform in E-multiplets.

No additional assumptions are needed beyond covariance w.r.t. $\mathcal{L}_0$.

Question: Can an E-multiplet mix inequivalent $\mathcal{L}_0$ irreps?

Theorem (analogue of Coleman–Mandula): If $\mathcal{A}$ obeys timeslice, additivity and local dynamical Lorentz invariance then

- $\mathcal{A}$ is covariant w.r.t. $\mathcal{S}$, universal cover of $\mathcal{L}_0$
- $[\mathcal{A}] \in H^2(\mathcal{S}, \mathrm{Aut}(\mathcal{A}))$ is trivial, so $E = \mathcal{L}_0 \times \mathrm{Aut}(\mathcal{A})$
- inequivalent $\mathcal{L}_0$ field multiplets are not mixed by E.

Local dynamical Lorentz invariance = triviality of r.c.e. for homotopically trivial local frame variation.

Corollary: If $n \geq 3$ then $\mathcal{A}$ has non-integer spin multiplets only if $Z(\mathrm{Aut}(\mathcal{A}))$ contains a copy of $\pi_1(\mathcal{L}_0)$ ($= \mathbb{Z}_2$ in $n \geq 4$).

- theories of observables only admit integer spin (trivial $\mathrm{Aut}(\mathcal{A})$)
- (for different reasons) the same is true of any theory pulled back to FLoc from Loc.

SPASs and dynamical locality CJF + Verch 2012

Contrary to intuition, a locally covariant theory need not represent the same physics in all spacetimes (SPASs).

Fact: There are non-constant functors $\mathcal{B} : \mathsf{Loc} \to \mathsf{Phys}^{\mathsf{Loc}}$. Any such $\mathcal{B}$ induces a 'diagonal theory' $\mathcal{B}_\Delta : \mathsf{Loc} \to \mathsf{Phys}$.

$$\begin{tikzcd} & \mathcal{B}(\boldsymbol{M})(\boldsymbol{N}) \arrow[dr, "\mathcal{B}(\psi)_{\boldsymbol{N}}"] & \\ \mathcal{B}_\Delta(\boldsymbol{M}) = \mathcal{B}(\boldsymbol{M})(\boldsymbol{M}) \arrow[ur, "\mathcal{B}(\boldsymbol{M})(\psi)"] \arrow[rr, dashed, "\mathcal{B}_\Delta(\psi)"] \arrow[dr, "\mathcal{B}(\psi)_{\boldsymbol{M}}"'] & & \mathcal{B}(\boldsymbol{N})(\boldsymbol{N}) = \mathcal{B}_\Delta(\boldsymbol{N}) \\ & \mathcal{B}(\boldsymbol{N})(\boldsymbol{M}) \arrow[ur, "\mathcal{B}(\boldsymbol{N})(\psi)"'] & \end{tikzcd}$$

$\mathcal{B}_\Delta$ agrees with $\mathcal{B}(\boldsymbol{M})$ in $\boldsymbol{M}$, but agrees with $\mathcal{B}(\boldsymbol{N})$ in $\boldsymbol{N}$.

$\mathcal{B}_\Delta$, $\mathcal{B}(\boldsymbol{M})$ and $\mathcal{B}(\boldsymbol{N})$ cannot each represent SPASs if $\mathcal{B}(\boldsymbol{M}) \ncong \mathcal{B}(\boldsymbol{N})$.

SPASs and dynamical locality CJF + Verch 2012

Contrary to intuition, a locally covariant theory need not represent the same physics in all spacetimes (SPASs).

It is difficult to give an intensive definition of SPASs and there may be many possible answers. We take an extensive approach.

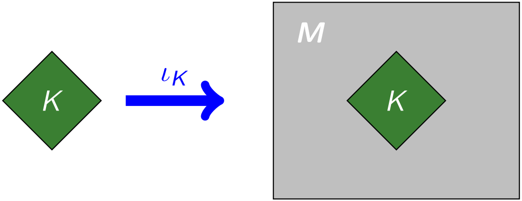
A collection $\mathfrak{T}$ of theories describes a coherent notion of SPASs if each subtheory embedding $\alpha : \mathcal{A} \dot{\to} \mathcal{B}$ with $\mathcal{A}, \mathcal{B} \in \mathfrak{T}$ is either an isomorphism in all spacetimes or in none.

An example of a coherent notion of SPASs may be given based on local physical content.

SPASs and dynamical locality (ctd)

Consider the local physical content of region K within $\boldsymbol{M}$

Kinematic description: $\mathcal{A}^{\text{kin}}(\boldsymbol{M}; K) = \mathcal{A}(\iota_K)(\mathcal{A}(\boldsymbol{M}|_K))$



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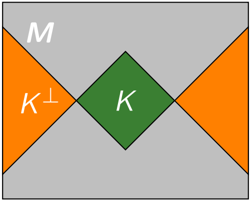
Dynamical description using relative Cauchy evolution:

$$\mathcal{A}^{\mathrm{dyn}}(\boldsymbol{M}; K) = \{A \in \mathcal{A}(\boldsymbol{M}) : \mathrm{rce}_{\boldsymbol{M}}[h]A = A \text{ for all } h \text{ supp in } K^{\perp}\}$$

If these descriptions agree, the theory is called dynamically local.



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Dynamical description using relative Cauchy evolution:

$$\mathcal{A}^{\mathrm{dyn}}(\boldsymbol{M}; K) = \{A \in \mathcal{A}(\boldsymbol{M}) : \mathrm{rce}_{\boldsymbol{M}}[h]A = A \text{ for all } h \text{ supp in } K^{\perp}\}$$

If these descriptions agree, the theory is called dynamically local.

In familiar models, dynamical locality holds except in cases of (broken?) gauge symmetry. CJF, Verch, Ferguson, Lang, Schenkel
Scalar field (+ external sources), extended Wick algebra, Dirac, e.m.

Theorem: The dyn. loc. theories form a coherent notion of SPASs.

Application: model-independent proof that there is no local and covariant way to choose a preferred state in each spacetime.

Conclusion

Local covariance is a mathematically natural and rich paradigm for QFT in CST, with numerous successes to its name.

It is conceptually clear, and allows the discussion of both foundational issues and physical applications in a way that would be cumbersome by other means.

Happy Birthday, Klaus! We look forward to many more insights and developments to come.