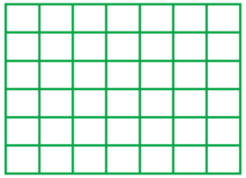
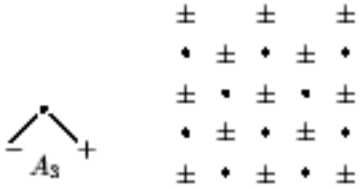
Operator Algebras: Impact of AQFT on Subfactors and K-theory

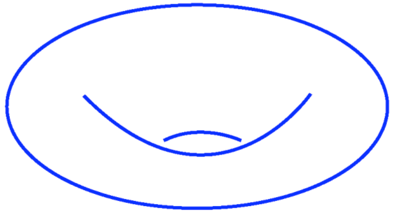
DEE, Terry Gannon
arXiv:0807.37591[math.KT]
Comm Number Thy & Phys
to appear

DEE, Mathew Pugh
arXiv:0906.431[math.OA]
Comm Math Phys
to appear

David E Evans
Cardiff

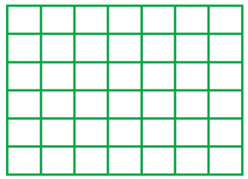
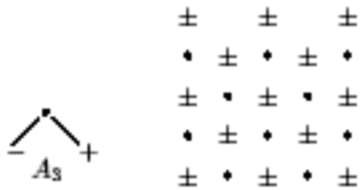
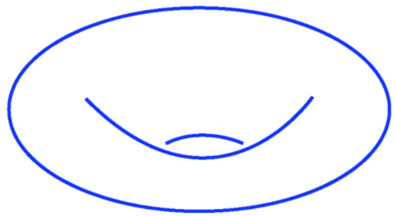






Ising $H(\sigma) \; = -\Sigma_{\alpha,\beta\ \mathrm{n.n}} J\, \sigma_\alpha\, \sigma_\beta$

$Z = \Sigma_\sigma \exp(-H(\sigma)) = \Sigma\ \Pi$ weights

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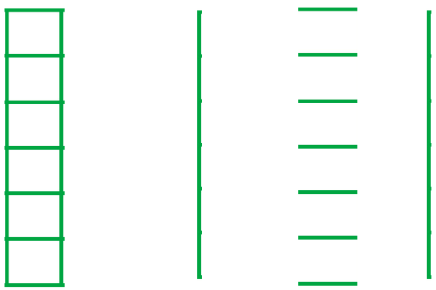
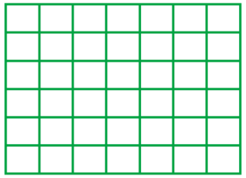
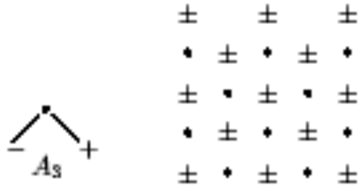
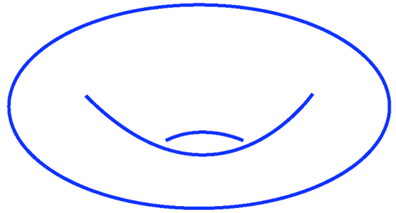
$T = V^{1/2}\, W\, V^{1/2} = e^{-\mathcal{H}}$



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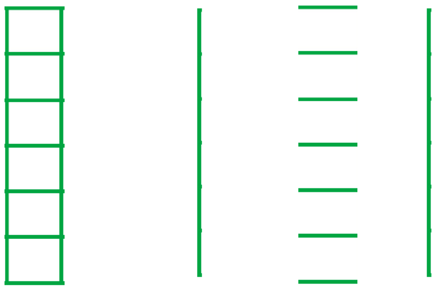

$V = \exp K\, \Sigma\, \sigma^x_j\, \sigma^x_{j+1}$ $\qquad$ $W = \exp L^*\, \Sigma\, \sigma^z_j$

$\sigma^x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\qquad$ $\sigma^z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$C\{+,-\}^{\mathbf{Z}^2} \;=\; \otimes_{\mathbf{Z}^2}(\mathbf{C}^2) \quad\to\quad M_2 \otimes M_2 \otimes M_2 \otimes M_2 \otimes \ldots. \text{ Pauli}$$

$$\mu(F) \;=\; \varphi_\mu(F_\beta) \qquad \alpha_t \;=\; T^{-it}(\;\;)T^{it} \;=\; \mathrm{Ad}\, e^{i\mathcal{H}t}$$

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$$\varphi_{\text{infinite}} = \otimes_{\mathbf{N}}\ \omega_{\binom{1/2^{1/2}}{1/2^{1/2}}} \qquad \omega_\xi A = \langle A\xi,\xi\rangle$$

$$\varphi_0^- = \otimes_{\mathbf{N}}\ \omega_{\binom{0}{1}}$$

$$\varphi_0^+ = \varphi_{\text{infinite}}\ \nu$$

Araki - Evans
Evans - Lewis

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Araki - Evans
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$$\sigma^x_j\, \sigma^x_{j+1} \quad \leftrightarrow \quad \sigma^z_j$$

$$\nu\, \sigma^x_j\, \sigma^x_{j+1} = \sigma^z_{j+1}$$
$$\nu\, \sigma^z_j = \sigma^x_j\, \sigma^x_{j+1}$$
$$\nu^2 = \text{shift on even algebra}$$

$$\nu\, \sigma^x_j = \sigma^z_1\, \sigma^z_2 \ldots \sigma^z_j$$

$$\nu^2\, \sigma^x_j = \sigma^x_1\, \sigma^x_{j+1}$$

$$M_2 \otimes M_2 \otimes M_2 \otimes M_2 \otimes \ldots\ldots \quad \subset \quad \text{Cuntz algebra } \mathcal{O}_2 = C^*(s_+, s_-)$$

$$s_+ s_+^* + s_- s_-^* = 1$$

$$\nu(s_+ + \sigma s_-)/2^{1/2} = s_+ s_\sigma s_\sigma^* + s_- s_{-\sigma} s_{-\sigma}^* \qquad \sigma = + \text{ or } -$$

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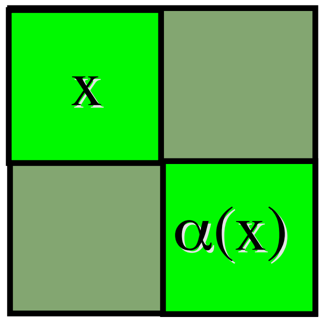
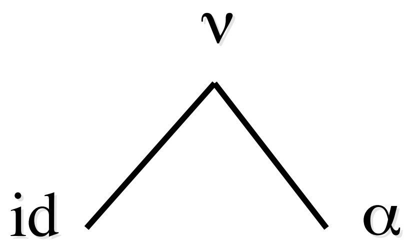
$$\nu^2(s_\sigma) = s_+ s_\sigma s_+^* + s_- s_{-\sigma} s_-^*$$

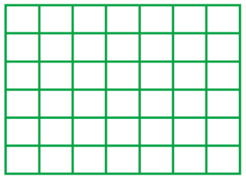
$$\nu^2(x) = s_+ x s_+^* + s_- \alpha(x) s_-^*$$

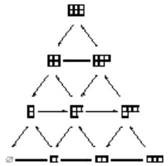


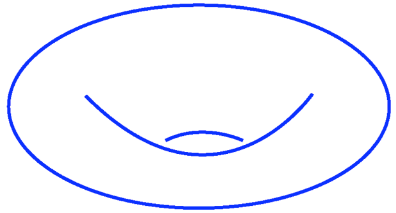
$$\alpha : s_+ \leftrightarrow s_- \qquad \nu^2 = \mathrm{id} + \alpha$$

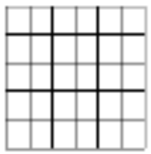
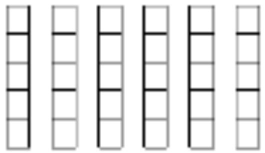
$$\alpha\nu = \nu$$

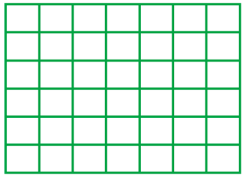
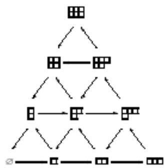
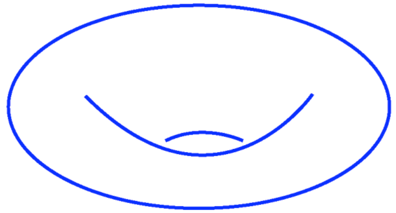


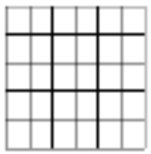
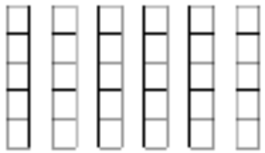




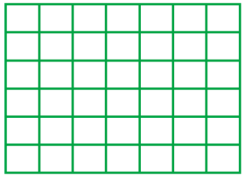
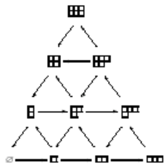
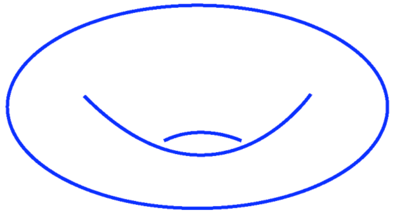


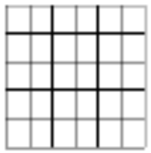
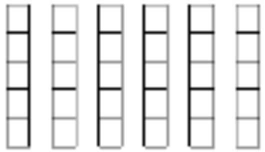
$Z =$  $=$  $= \mathrm{tr}T^N$

$Z =$  $=$  $= \mathrm{tr}T^N$

$$\rightarrow \quad \mathrm{tr}\, e^{2\pi i\tau(L_o - c/24)}\, e^{-2\pi i\bar{\tau}(\bar{L}_o - c/24)} \qquad \chi_\lambda(\tau) = \mathrm{tr}_\lambda\, e^{2\pi i\tau(L_o - c/24)}$$

$Z =$  $=$  $= \mathrm{tr}T^N$

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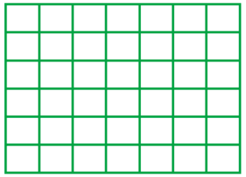
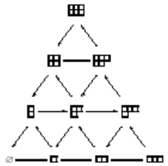
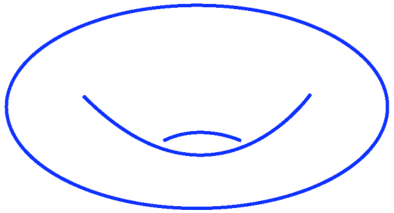
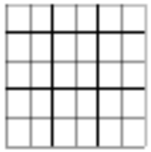
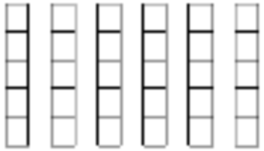
$= \quad \Sigma\, Z_{\lambda\mu}\, \chi_\lambda(\tau)\, \chi_\mu(\tau)^* = Z(\tau) = Z((a\tau + b)/(c\tau+d))$

$Z =$  $=$  $= \mathrm{tr}T^N$

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$Z = [Z_{\lambda\mu}] \quad \in \quad SL(2, \mathbf{Z})'$

$Z_{\lambda\mu} \in \ 0,1,2,3,\ldots. \qquad Z_{00} = 1$

Ising model $\lambda \in \{\bullet, +, -\}$

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$$L_0 = \sum_{r \in \mathbf{N} - 1/2} r\, g_r^* g_r \longrightarrow \chi_{\pm}$$

g_a Fermions

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$$q = e^{2\pi i \tau}$$

$$\chi_+ + \chi_- = q^{-1/48} \prod_{n \in \mathbf{N}} (1 \pm q^{n-1/2})$$

$$\chi = \mathrm{tr}\, q^{L_0 - c/24}$$

$$\chi_{\bullet} = q^{1/24} \prod_{n \in \mathbf{N}} (1 + q^n)$$

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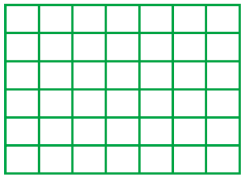
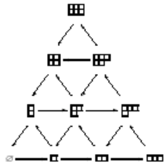
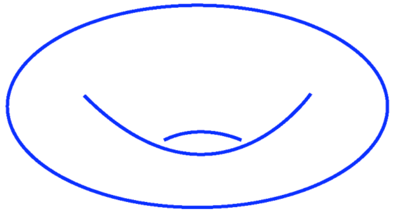
$$\tau \longrightarrow \tau + 1 \qquad T = \begin{pmatrix} e^{-\pi i/24} & & \\ & e^{-\pi i/12} & \\ & & e^{\pi i 23/24} \end{pmatrix}$$

$$\tau \longrightarrow -1/\tau \qquad S = \begin{pmatrix} 1 & 2^{1/2} & 1 \\ 2^{1/2} & 0 & -2^{1/2} \\ 1 & -2^{1/2} & 0 \end{pmatrix}$$

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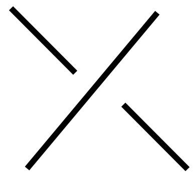
λ endomorphism N type III_1 factor: $\lambda\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu\, \nu$

$\lambda\mu = \mathrm{Ad}\, u(\lambda,\mu)\ \ \mu\lambda \qquad\qquad u(\lambda,\mu) =$ 

$\begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} \quad \rightarrow \quad [S_{\lambda\mu}]$ 

$\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} \quad \rightarrow \quad [T_{\lambda\mu}]$ 

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Braided systems of endomorphisms:

- Loop groups SU(2), .., SU(N) etc **Wasserman**

  $\pi_\lambda(L_I SU(n))'' \quad \subset \quad \pi_\lambda(L_{I'} SU(n))'$

  $\lambda=0$: $\quad N = N \quad III_1$ factor , $\quad \lambda N \subset N$

- Quantum double of finite group, Haagerup subfactor etc

  Ocneanu, Izumi

$\Sigma\ \chi_\lambda\, \chi_\lambda{}^*$

$\Sigma\ Z_{\lambda\mu}\, \chi_\lambda\, \chi_\mu{}^*$ $\Sigma\ \chi_\tau\, \chi_{\sigma\tau}{}^*$

A on N $\subset$ B on M

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$\Sigma\ b_{\tau\lambda}\ \chi_\lambda\ =\ \chi_\tau$ ${}_N M_N$ = sum of λ's

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A on	N	$\subset$	M
			B on

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$$= \Sigma\ (\boldsymbol{\Sigma}\ \mathbf{b}_{\tau\lambda}\mathbf{b}_{\sigma\tau\mu})\, \chi_\lambda\, \chi_\mu{}^* = \Sigma\ \mathbf{Z}_{\lambda\mu}\, \chi_\lambda\, \chi_\mu{}^*$$

$$Z_{\lambda\mu} = \Sigma\ b_{\tau\lambda} b_{\sigma\tau\mu}$$

$\Sigma\ \chi_\lambda \chi_\lambda^{\ *}$

$\Sigma\ Z_{\lambda\mu} \chi_\lambda \chi_\mu^{\ *}$ $\Sigma\ \chi_\tau \chi_{\sigma\tau}^{\ *}$

N $\subset$ M

A on B on

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$$Z_{\lambda\mu} = \Sigma\ b_{\tau\lambda} b_{\sigma\tau\mu}$$

${}_N M_N = \Sigma\ Z_{0,\lambda}\ \lambda$ local

$\Sigma\ \chi_\lambda \chi_\lambda{}^*$

$\Sigma\, Z_{\lambda\mu} \chi_\lambda \chi_\mu{}^*$ $\Sigma\ \chi_\tau \chi_{\sigma\tau}{}^*$

$N \subset M$

A on N B on M

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$Z_{\lambda\mu} = \Sigma\ b_{\tau\lambda} b_{\sigma\tau\mu}$ $N \subset M_\pm \subset M$ **Bockenhauer-Evans**

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$$\Sigma\ \chi_\lambda\, \chi_\lambda{}^*$$

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N	$\subset$ M
A on	B on

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$Z_{\lambda\mu} = \Sigma\ b_{\tau\lambda} b_{\sigma\tau\mu}$ $\qquad$ $N \subset M_\pm \subset M$ **Bockenhauer-Evans**

${}_N M_N = \Sigma\, Z_{0,\lambda}\, \lambda$ $\quad$ local

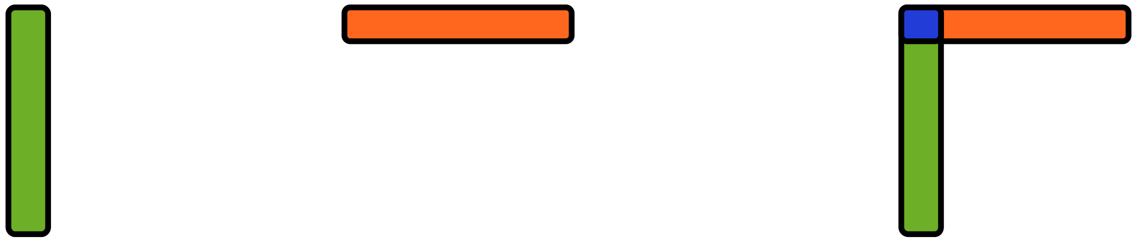
$$\Sigma\ \tau \otimes \sigma\tau^{opp}$$

$$N(I) \otimes N(J)^{opp} \subset M_+(I) \otimes M_-(J)^{opp} \subset B(I \times J)$$

$$\Sigma\, Z_{\lambda,\mu}\, \lambda \otimes \mu^{opp}$$

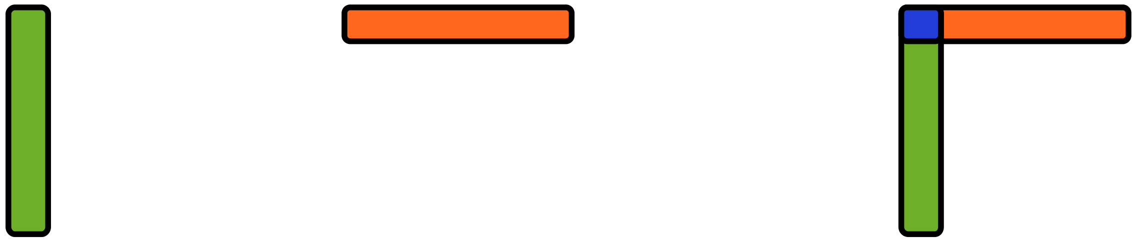
B-E+Rehren

$N \subset M$ - induce λ to α_λ using braiding and opposite braiding



$$A^+ \cap A^- = B$$

$N \subset M$ - induce λ to α_λ using braiding and opposite braiding



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- $Z_{\lambda\mu} = \langle {}^+\alpha_\lambda, {}^-\alpha_\mu \rangle$ is a modular invariant

Bockenhauer-Evans-Kawahigashi, Feng Xu, Ocneanu

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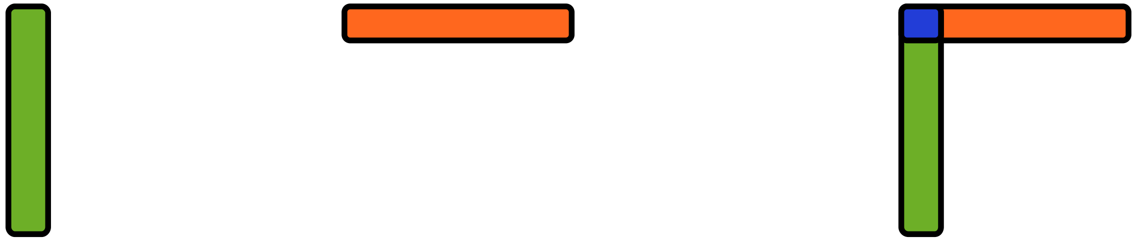

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Bockenhauer-Evans-Kawahigashi, Feng Xu, Ocneanu

- N-M sectors from $\iota\lambda$

$\iota: N \subset M, \quad \lambda \in$ N-N system

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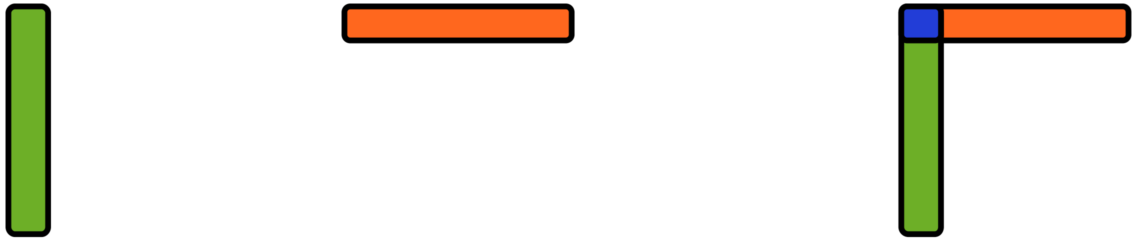

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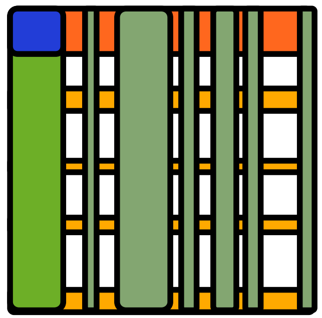
Bockenhauer-Evans-Kawahigashi,
Feng Xu, Ocneanu

- N-M sectors from $\iota\lambda$

$\iota: N \subset M, \quad \lambda \in$ N-N system

- M-M sectors from $\iota \, \lambda \, \bar{\iota}$

CIZ graph for ZZ^*



$$\lambda\mu = \Sigma_\nu N_{\lambda\mu}{}^{\nu}\,\nu$$

$$N_\lambda N_\mu = \Sigma_\nu N_{\lambda\mu}{}^{\nu} N_\nu \qquad\qquad N_\lambda = \Sigma_\kappa\, S_{\lambda\kappa}/\, S_{0\kappa}\, |S_\kappa\rangle\langle S_\kappa|$$

$$N_\lambda = [N_{\lambda\mu}{}^{\nu}]_{\mu\nu}$$

Verlinde algebra

$$\lambda\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu \nu$$

$$N_\lambda N_\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu N_\nu \qquad N_\lambda = \Sigma_\kappa \; S_{\lambda\kappa}/ \; S_{0\kappa} \; |S_\kappa\rangle\langle S_\kappa|$$

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Verlinde algebra

G action of N-N on N-M $\quad G_\lambda = \Sigma_\kappa \; S_{\lambda\kappa}/ \; S_{0\kappa} \; |\psi_\kappa\rangle\langle\psi_\kappa|$

$$G_\lambda = [G_{\lambda a}{}^b]_{ab} \qquad G_\lambda G_\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu G_\nu$$

$$\lambda\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu \nu$$

$$N_\lambda N_\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu N_\nu \qquad N_\lambda = \Sigma_\kappa S_{\lambda\kappa}/ S_{0\kappa} |S_\kappa\rangle\langle S_\kappa|$$

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Verlinde algebra

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$$G_\lambda = [G_{\lambda a}{}^b]_{ab} \qquad G_\lambda G_\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu G_\nu$$

$\boldsymbol{\sigma(G_\lambda) = \{S_{\lambda\mu}/S_{0\lambda} : \text{multiplicity } Z_{\lambda\lambda}\}}$ **Bockenhauer-Evans-Kawahigashi**

$$\lambda\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu \nu$$

$N_\lambda N_\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu N_\nu$ $\qquad$ $N_\lambda = \Sigma_\kappa S_{\lambda\kappa}/ S_{0\kappa} |S_\kappa\rangle\langle S_\kappa|$

$N_\lambda = [N_{\lambda\mu}{}^\nu]_{\mu\nu}$ $\qquad$ **Verlinde algebra**

G action of N-N on N-M $\qquad$ $G_\lambda = \Sigma_\kappa S_{\lambda\kappa}/ S_{0\kappa} |\psi_\kappa\rangle\langle\psi_\kappa|$

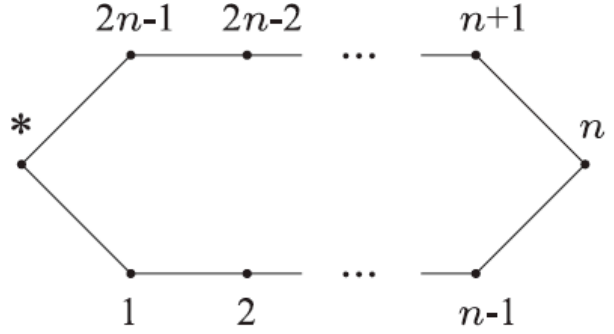
$G_\lambda = [G_{\lambda a}{}^b]_{ab}$ $\qquad$ $G_\lambda G_\mu = \Sigma_\nu N_{\lambda\mu}{}^\nu G_\nu$

$\boldsymbol{\sigma(G_\lambda) = \{S_{\lambda\mu}/S_{0\lambda} : \text{multiplicity } Z_{\lambda\lambda}\}}$ $\qquad$ **Bockenhauer-Evans-Kawahigashi**

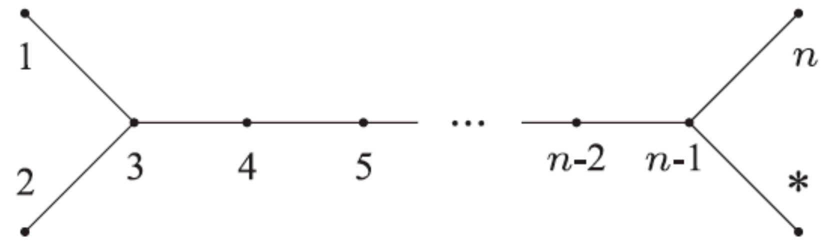
nimrep $\qquad$ gives ADE classification for SU(2) $\qquad$ **Capelli-Itzykson-Zuber**

nonnegative **i**nteger **m**atrix **rep** $\quad$ of original Verlinde algebra

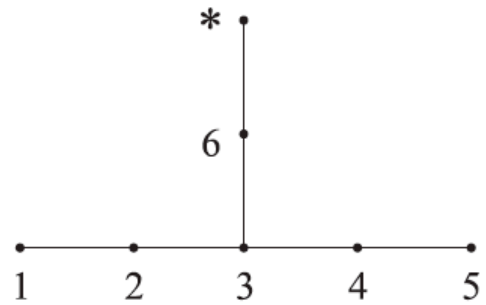
$A_{2n}^{(1)}$



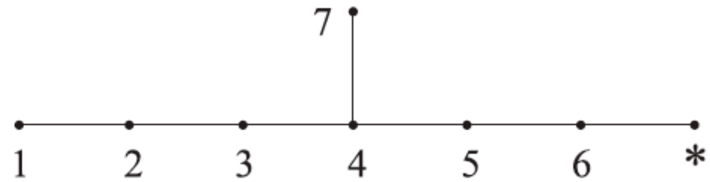
$D_n^{(1)}$



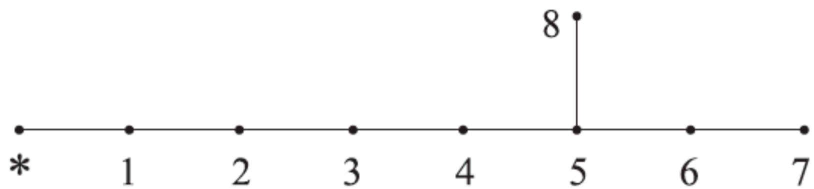
$E_6^{(1)}$



$E_7^{(1)}$

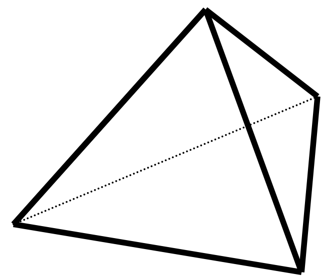


$E_8^{(1)}$



A cyclic
D dihedral

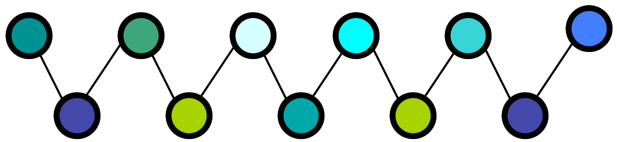
$\mathbf{E_6}$ tetrahedral
$\mathbf{E_7}$ octahedral
$\mathbf{E_8}$ icosahedral



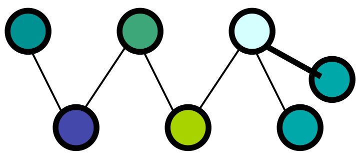
subgroups of SU(2)

ADE classification $SU(2)_{10}$

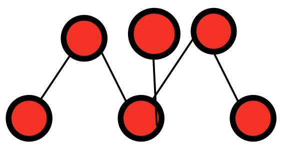
eigenvalues $2 \cos \pi\, \lambda/12$ $= S_{1\lambda}/S_{0\lambda}$ λ exponents



0 1 2 3 4 5 6 7 8 9 10



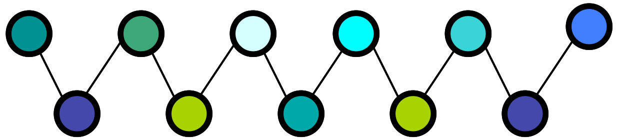
0 2 4 6 8 10 5



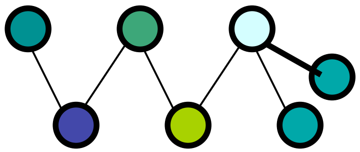
0 3 4 6 7 10

$SU(2)_{10}$ A_{11} D_7 E_6 **Capelli-Itzykson-Zuber**

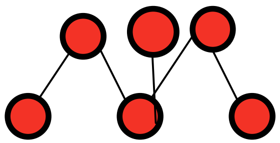
$$Z_A = |\chi_0|^2 + |\chi_1|^2 + |\chi_2|^2 + |\chi_3|^2 + \ldots\ldots + |\chi_8|^2 + |\chi_9|^2 + |\chi_{10}|^2$$



$$Z_D = |\chi_0|^2 + |\chi_{10}|^2 + |\chi_2|^2 + |\chi_8|^2 + |\chi_4|^2 + |\chi_6|^2 + |\chi_5|^2 + \chi_1\chi_9^* + \chi_9\chi_1^* + \chi_3\chi_7^* + \chi_7\chi_3^*$$



$$Z_E = |\chi_0 + \chi_6|^2 + |\chi_4 + \chi_{10}|^2 + |\chi_3 + \chi_{12}|^2$$

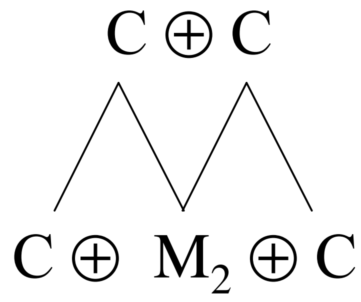


$$\sigma(E_6) = \{S_{\lambda\mu}/S_{0\lambda} : \lambda = \mathbf{0, 6, 4, 10, 3, 12}\}$$

$$(M_2 \otimes M_2 \otimes M_2 \otimes M_2 \otimes \ldots\ldots)^T \qquad \otimes_N \mathrm{Ad}\begin{pmatrix} t & \\ & t^{-1} \end{pmatrix}$$

$$M_2 \qquad \begin{pmatrix} t & \\ & t^{-1} \end{pmatrix}$$

$$M_2 \otimes M_2 \qquad \begin{pmatrix} t & \\ & t^{-1} \end{pmatrix} \otimes \begin{pmatrix} t & \\ & t^{-1} \end{pmatrix} = \begin{matrix} t^2 & & & \\ & 1 & & \\ & & 1 & \\ & & & t^{-2} \end{matrix}$$



$$(M_2 \otimes M_2 \otimes M_2 \otimes M_2 \otimes \ldots\ )^T \qquad \otimes_N \mathrm{Ad} \begin{pmatrix} t & \\ & t^{-1} \end{pmatrix}$$

$M_2 \qquad \begin{pmatrix} t & \\ & t^{-1} \end{pmatrix}$

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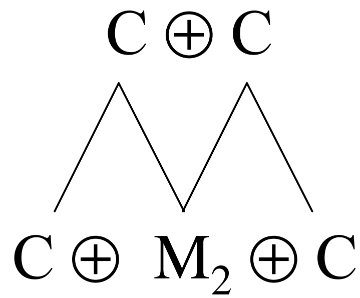

$K_0(\otimes_N M_2)^T = Z[t] \qquad t^m(1-t)^n$

$P(t) > 0$ on $(0,1)$ **Renault**

$$(M_2 \otimes M_2 \otimes M_2 \otimes M_2 \otimes \ldots.\ )^T \qquad \otimes_N \mathrm{Ad}\begin{pmatrix} t & \\ & t^{-1} \end{pmatrix}$$

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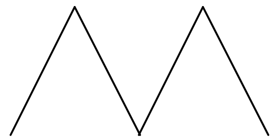
$C \oplus C$



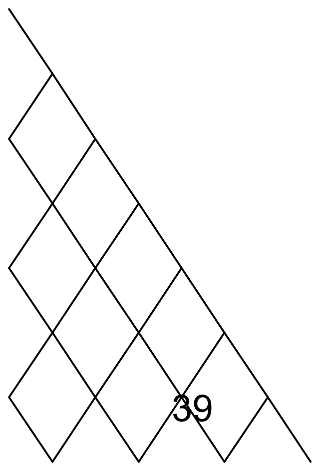
$C \oplus\ M_2 \oplus C$

$K_0(\otimes_N M_2)^T = Z[t] \qquad t^m(1\text{-}t)^n$

$P(t) > 0$ on $(0,1)$ **Renault**

$K_0(\otimes_N M_2)^{SU(2)} = Z[t] \qquad P_i \qquad tP_i = P_{i-1} + P_{i+1}$

$P(t) > 0$ on $(0,1/4)$ **A Wassermann**



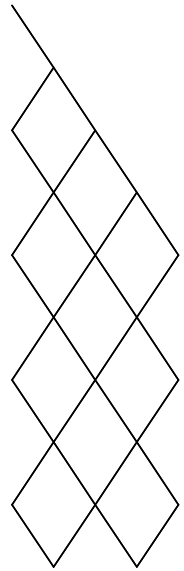
$K_0(\otimes_N M_2)^{SU(2)} = Z[t] = R_{SU(2)}$ $\qquad P_i \qquad tP_i = P_{i-1} + P_{i+1}$

Verlinde algebra at ∞ level

$K_0(\otimes_N M_2)^{SU(2)} = Z[t] = R_{SU(2)}$ $\quad P_i \quad$ $tP_i = P_{i-1} + P_{i+1}$

Verlinde algebra at ∞ level

$K_0(\otimes_N M_2)^{SU_q(2)} = R_{SU(2)} / \langle P_k \rangle$ **Evans-Gould**



$q = e^{i\pi/(k+2)}$ Verlinde algebra at level k

$P((2 \cos \pi/(k+2))^{-2}) > 0$

$K_0(\otimes_N M_2)^{SU(2)} = Z[t] = R_{SU(2)}$ P_i $tP_i = P_{i-1} + P_{i+1}$

Verlinde algebra at ∞ level

$K_0(\otimes_N M_2)^{SU_q(2)} = R_{SU(2)} / \langle P_k \rangle$ **Evans-Gould**

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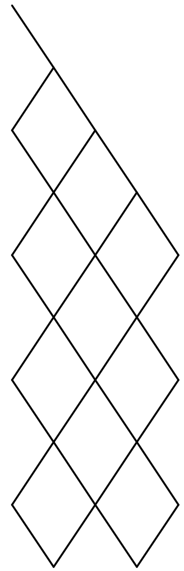
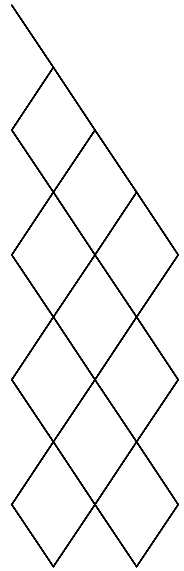


$K_0^{SU_q(2)}(\otimes_N M_2)$ equivariant K-theory

$K_0(\otimes_N M_2)^{SU(2)} = Z[t] = R_{SU(2)}$ $\quad P_i \quad tP_i = P_{i-1} + P_{i+1}$

Verlinde algebra at ∞ level



$K_0(\otimes_N M_2)^{SU_q(2)} = R_{SU(2)} /\langle P_k\rangle$ **Evans-Gould**

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$K_0^{SU_q(2)} (\otimes_N M_2)$ equivariant K-theory

${}^{\text{twist}}K_0^{SU(2)} (SU(2))$ **Freed Hopkins Teleman**

Verlinde algebra

${}^{\tau}K^0_G(G)$ G on G by conjugation

Verlinde algebra

${}^{\tau}K^0_G(G)$ G on G by conjugation

G finite: $C(G) = C^G$ $K^0(G) = Z^G$

Verlinde algebra

${}^{\tau}K^0_G(G)$ G on G by conjugation

G finite: $C(G) = C^G$ $K^0(G) = Z^G$

G abelian

$K^0_G(G) = Z^G \otimes R(G)$ as $C(G) \times G = C(G) \otimes C^*(G)$

irreducibles/primary fields (g,π)

$G, G^\wedge$

Verlinde algebra

${}^{\tau}K^0_G(G)$ G on G by conjugation

G finite: $C(G) = C^G$ $K^0(G) = Z^G$

G abelian

$K^0_G(G) = Z^G \otimes R(G)$ as $C(G)\times G = C(G)\otimes C^*(G)$

irreducibles/primary fields (g,π)

$G, G^\wedge$

$N \subset N\times G$ doubled: $N\times G \subset N\times(G\times G)$

$G \subset G\times G \approx G$ on G

$\nu_g \nu_h = \mathrm{Ad}\, u(g,h)\, \nu_{gh}$ $\qquad \nu_g \in \mathrm{Aut}(N) \qquad g \in G$ finite group

$(\nu_g \nu_h)\, \nu_k = \nu_g\, (\nu_h \nu_k)$

$u(g,h)u(gh,k) = \omega(g,h,k)\, \nu_g u(h,k)\, u(g,hk) \qquad \omega \in Z^3(G,T)$

$$\nu_g \nu_h = \mathrm{Ad}\, u(g,h)\, \nu_{gh} \qquad \nu_g \in \mathrm{Aut}(N) \qquad g \in G \text{ finite group}$$

$$(\nu_g \nu_h)\, \nu_k = \nu_g\, (\nu_h \nu_k)$$

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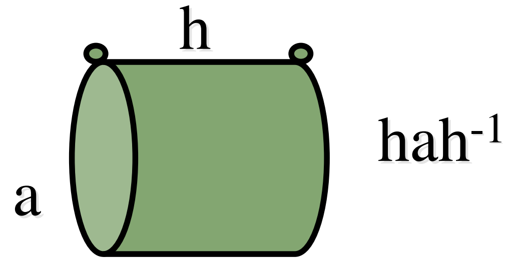
$$\mathrm{Hom}(\nu_h \nu_a, \nu_{hah^{-1}} \nu_h) = C\, u(hah^{-1}, h)\, u(h, a)^* =$$



$$\nu_g \nu_h = \mathrm{Ad}\, u(g,h)\, \nu_{gh} \qquad \nu_g \in \mathrm{Aut}(N) \qquad g \in G \text{ finite group}$$

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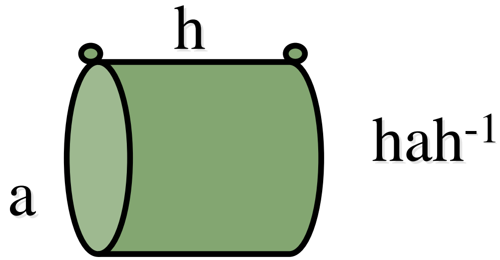


$$t(hah^{-1}, k)\, t(a, h) = w\, t(a, kh)$$

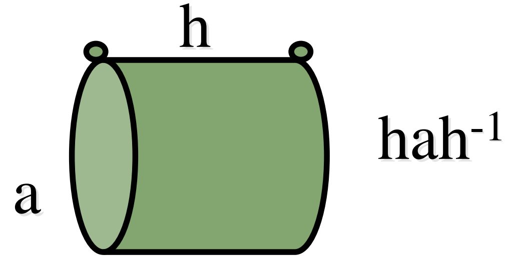
$$w = \omega(k, h, a)\omega^{-1}(k, hah^{-1}, h)\omega(khah^{-1}k^{-1}, k, h)$$

$$\nu_g \nu_h = \mathrm{Ad}\, u(g,h)\, \nu_{gh} \qquad \nu_g \in \mathrm{Aut}(N) \qquad g \in G \text{ finite group}$$

$$(\nu_g \nu_h)\, \nu_k = \nu_g\, (\nu_h \nu_k)$$

$$u(g,h)u(gh,k) = \omega(g,h,k)\, \nu_g u(h,k)\, u(g,hk) \qquad \omega \in Z^3(G,T)$$

$$\mathrm{Hom}(\nu_h \nu_a, \nu_{hah^{-1}} \nu_h) = C\, u(hah^{-1}, h)\, u(h, a)^* =$$



$$t(hah^{-1}, k)\, t(a, h) = w\, t(a, kh)$$

$$w = \omega(k, h, a)\omega^{-1}(k, hah^{-1}, h)\omega(khah^{-1}k^{-1}, k, h)$$

$$w \in H^2(G \text{ on } G,T) = H^2_G(G,T) \to H^3_G(G, Z))$$

$H^3(G,T)$ Connes-Jones	$\to$	$H^3_G(G, Z)$ Dixmier-Douady

$$\nu_g \nu_h = \mathrm{Ad}\, u(g,h)\, \nu_{gh} \qquad\qquad \nu_g \in \mathrm{Aut}(N)$$

$$(\nu_g \nu_h)\, \nu_k = \nu_g\, (\nu_h \nu_k)$$

$$u(g,h)u(gh,k) = \omega(g,h,k)\, \nu_g u(h,k)\, u(g,hk) \qquad \omega \in Z^3(G,T)$$

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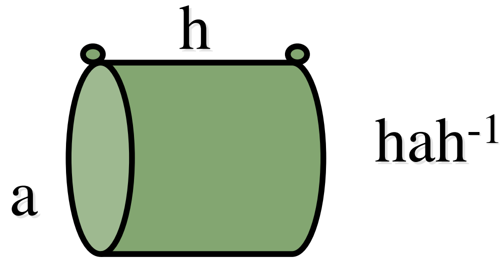
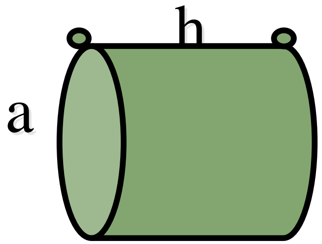


$V_a \otimes$



$\approx V_{hah^{-1}}$

${}^{\omega}K_G(G) \approx$ N-N sectors

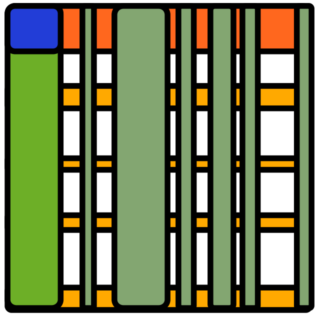
H^3 = level

Conformal embedding for finite groups

$$G\times G \xrightarrow{\alpha} G \xrightarrow{\pi} L$$

$$(a,b) \rightarrow ab^{-1}$$

$$H = \ker \pi\alpha \supseteq \Delta$$



Conformal embedding for finite groups

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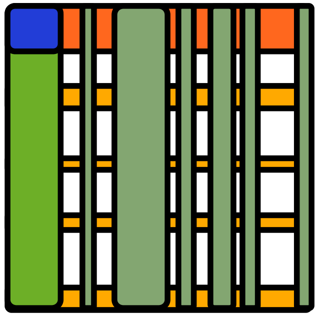
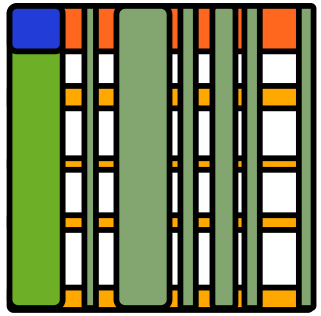


σ–restriction $\quad K^0{}_L(L) \to K^0{}_G(G)$

Conformal embedding for finite groups



$$G\times G \xrightarrow[\alpha]{} G \xrightarrow[\pi]{} L \qquad H = \ker \pi\alpha \supseteq \Delta$$

$$(a,b) \rightarrow ab^{-1}$$

σ–restriction $K^0{}_L(L) \rightarrow K^0{}_G(G)$

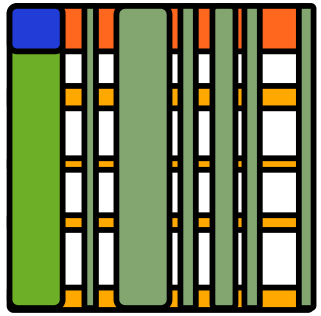
$$\underset{L,L^\wedge}{\sigma_{l,\psi}} = \sum_{g\in\pi^{-1}(l)} [g,\psi\pi] \qquad \Theta = \text{restriction of H-H trivial bundle to } \Delta\text{–}\Delta$$

Conformal embedding for finite groups

$$G\times G \xrightarrow[\alpha]{} G \xrightarrow[\pi]{} L$$

$$(a,b) \rightarrow ab^{-1}$$

$H = \ker \pi\alpha \supseteq \Delta$



σ–restriction $K^0{}_L(L) \rightarrow K^0{}_G(G)$

$$\underset{L,L^\wedge}{\sigma_{l,\psi}} = \sum_{g\in\pi^{-1}(l)} [g,\psi\pi]$$

Θ = restriction of H-H trivial bundle to Δ–Δ

$$\Theta \otimes_\Delta \lambda$$

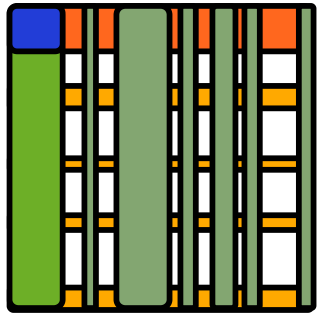
H H

Conformal embedding for finite groups

$$G\times G \xrightarrow{\alpha} G \xrightarrow{\pi} L$$

$$(a,b) \to ab^{-1}$$

$H = \ker \pi\alpha \supseteq \Delta$



σ–restriction $\quad K^0{}_L(L) \to K^0{}_G(G)$

$$\underset{L,L^\wedge}{\sigma_{l,\psi}} = \sum_{g\in\pi^{-1}(l)} [g,\psi\pi]$$

Θ = restriction of H-H trivial bundle to Δ–Δ

α–induction $\quad K^0{}_L(L) \to K^0{}_{H\text{-}H}(G\times G)$

$$\Theta \otimes_\Delta \lambda$$

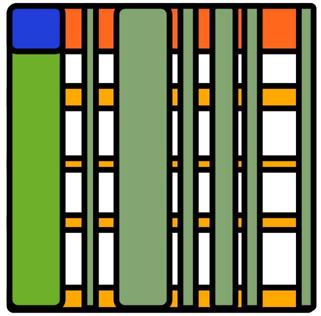
H $\qquad\qquad$ H

Conformal embedding for finite groups

$$G\times G \xrightarrow{\alpha} G \xrightarrow{\pi} L$$

$$(a,b) \to ab^{-1}$$

$H = \ker \pi\alpha \supseteq \Delta$



σ–restriction $\quad K^0_L(L) \to K^0_G(G)$

$$\sigma_{l,\psi} = \sum_{g\in\pi^{-1}(l)} [g,\psi\pi]$$
$L,L^\wedge$

Θ = restriction of H-H trivial bundle to Δ–Δ

$\Theta \otimes_\Delta \lambda$

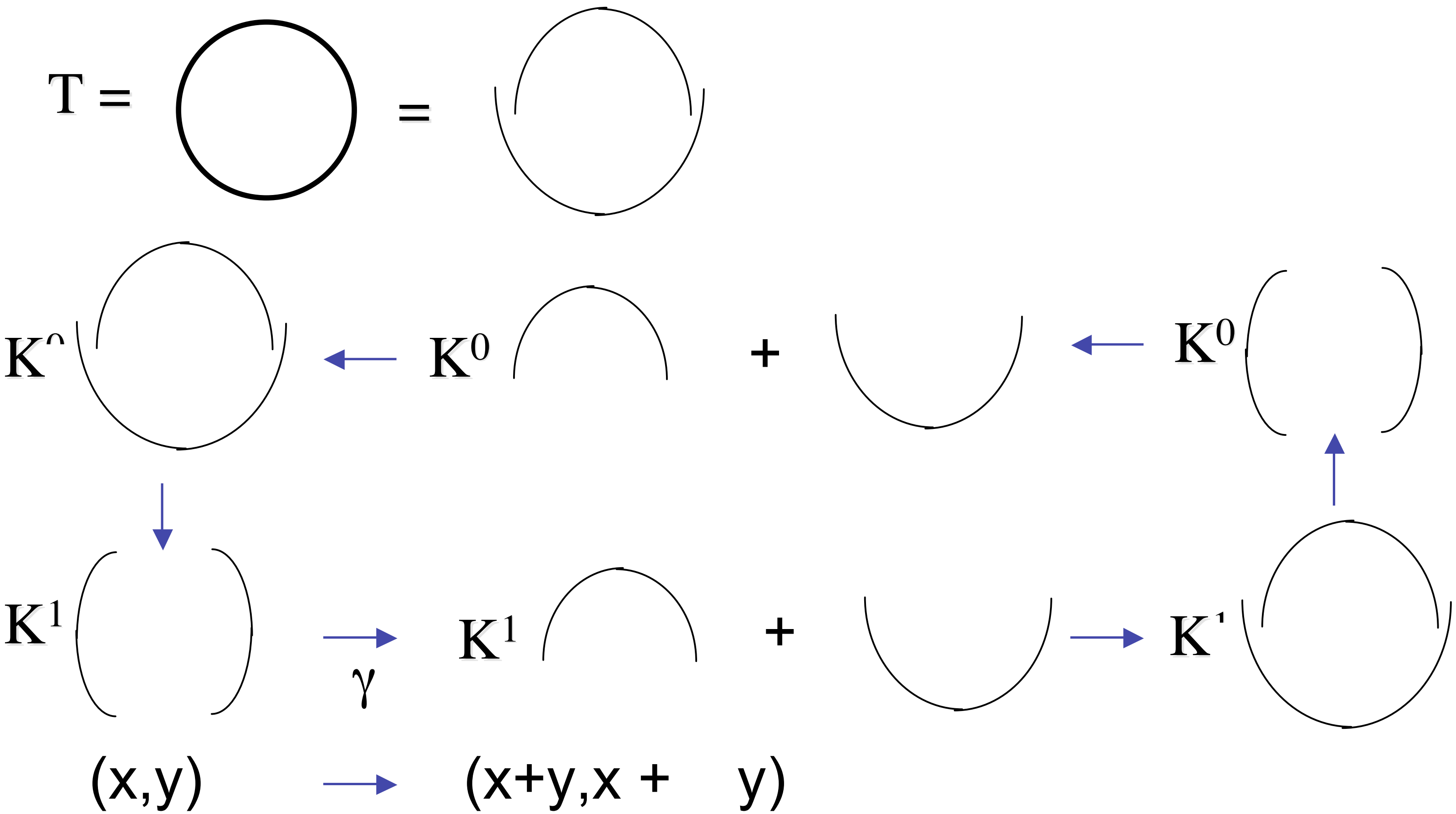
H $\qquad$ H

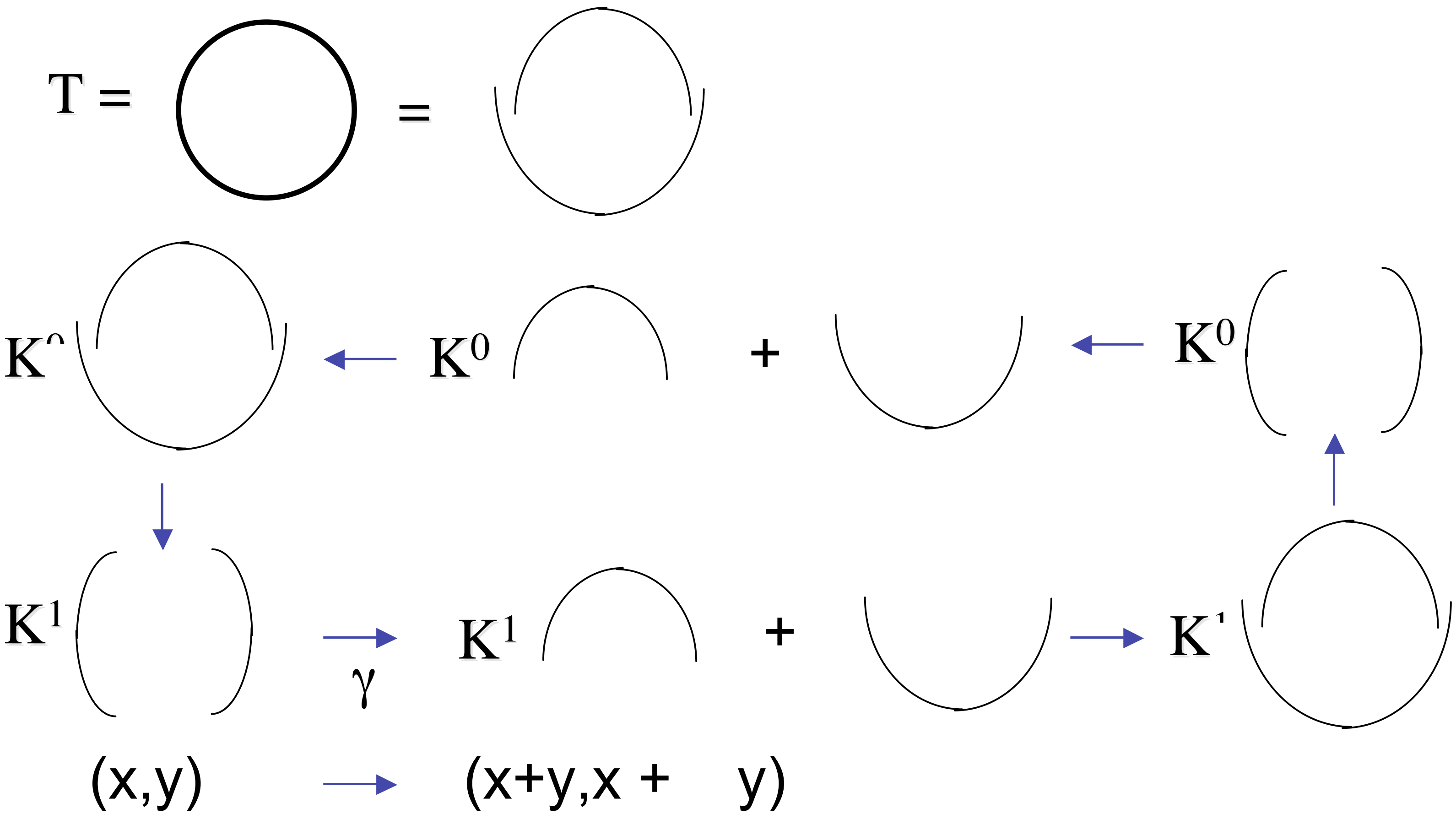
α–induction $\quad K^0_L(L) \to K^0_{H\text{-}H}(G\times G)$

$\alpha^+ : [a,\theta] \to [(a\times 1)H, (\theta\times 1)|H] \qquad K^0_{\Delta\text{-}H}(G\times G)$

$\alpha^- : [b,\psi] \to [(1\times b)H, (\psi\times 1)|H] \qquad K^0_{H\text{-}\Delta}(G\times G)$

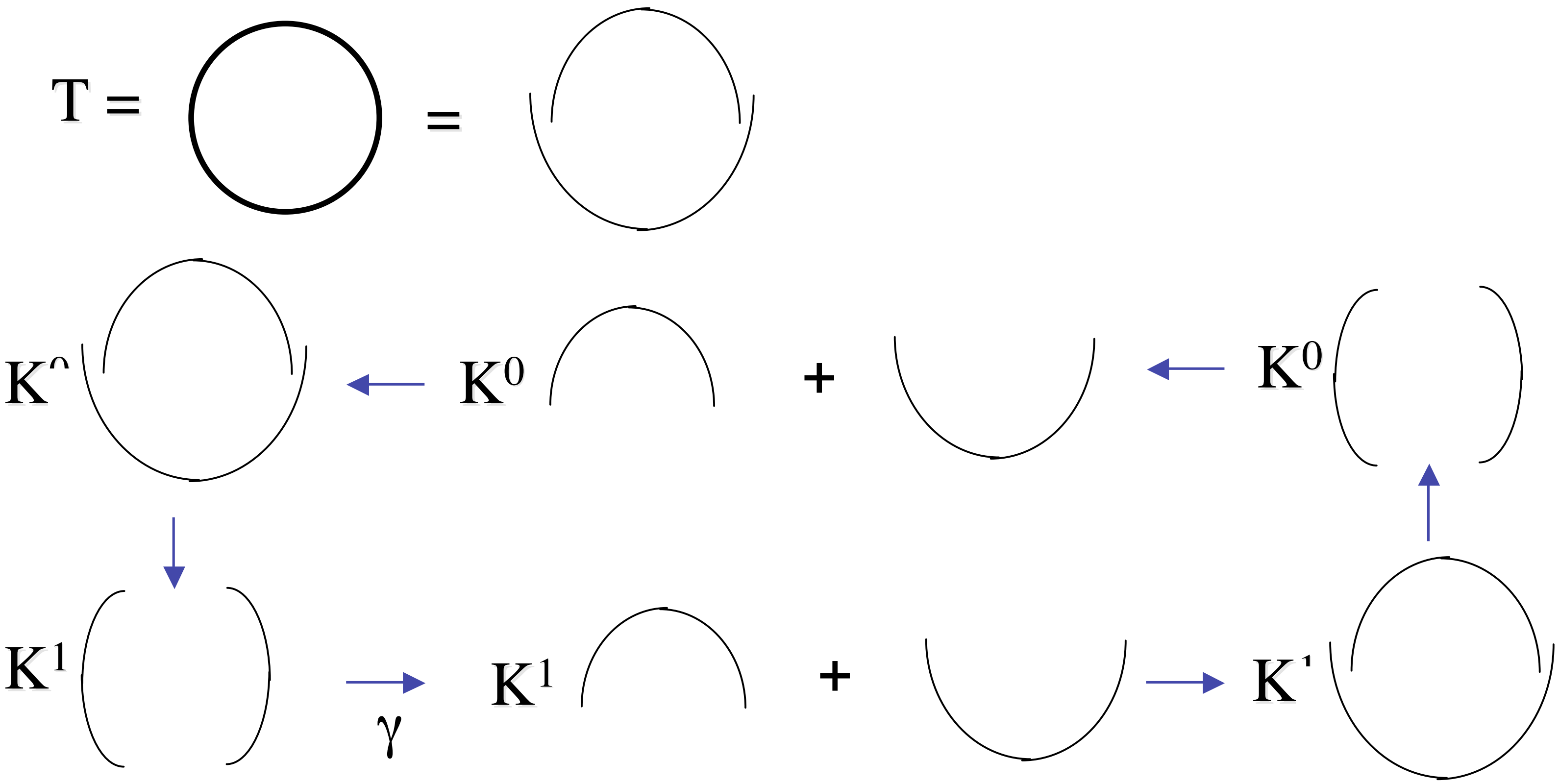
$\alpha^+ \cap \alpha^- \approx K^0_L(L)$


T =
=
K^0
K^0
+
K^0
K^1
γ
K^1
+
K^1
(x,y)
(x+y,x + y)



$$K^0(T) = \ker \gamma = Z \qquad K^1(T) = \operatorname{coker} \gamma = Z$$

$$K^0{}_T(T) = \ker \gamma = R(T) \qquad K^1{}_T(T) = \operatorname{coker} \gamma = R(T)$$

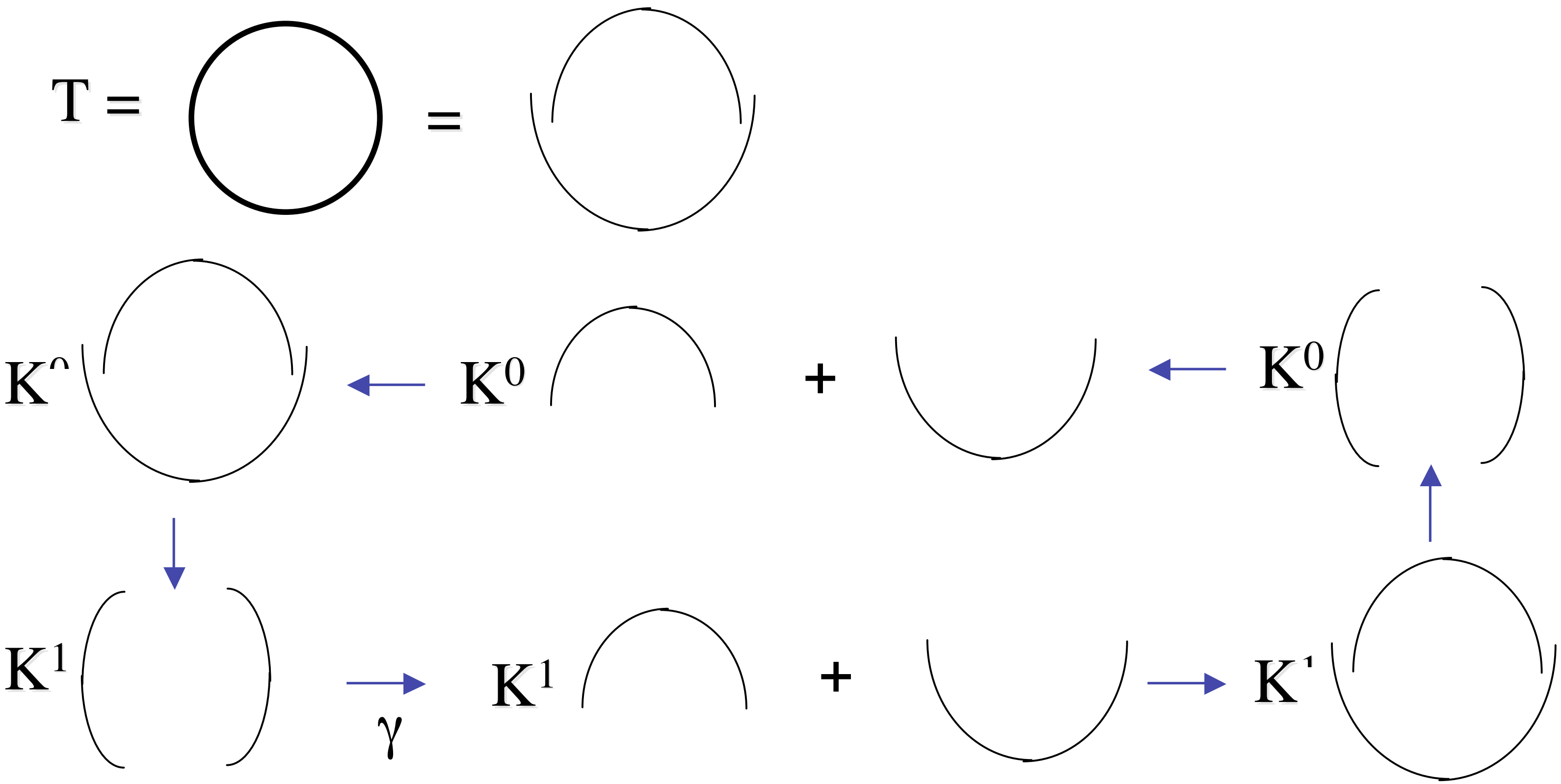


$$(x,y) \longrightarrow (x+y, x + a^k y)$$

$$K^0(T) = \ker \gamma = \mathbb{Z} \qquad K^1(T) = \operatorname{coker} \gamma = \mathbb{Z}$$

$$K^0_T(T) = \ker \gamma = R(T) \qquad K^1_T(T) = \operatorname{coker} \gamma = R(T)$$

$$^kK^0_T(T) = \ker \gamma = 0 \qquad ^kK^1_T(T) = \operatorname{coker} \gamma = R(T)/(1-a^k)R(T)$$



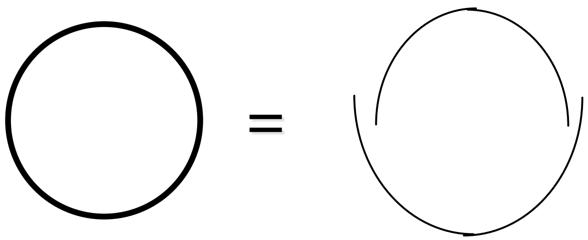
$(x,y) \longrightarrow (x+y, x + a^k y)$

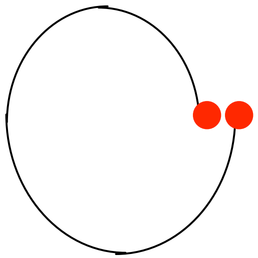
$K^0(T) = \ker\gamma = Z$ $\qquad$ $K^1(T) = \operatorname{coker}\gamma = Z$

$K^0{}_T(T) = \ker\gamma = R(T)$ $\qquad$ $K^1{}_T(T) = \operatorname{coker}\gamma = R(T)$

${}^kK^0{}_T(T) = \ker\gamma = 0$ $\qquad$ ${}^kK^1{}_T(T) = \operatorname{coker}\gamma = R(T)/(1-a^k)R(T)$

Twist in H^1 $\qquad$ $K^0 = 0$ $\qquad$ $K^1 = Z/2$

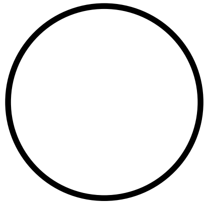
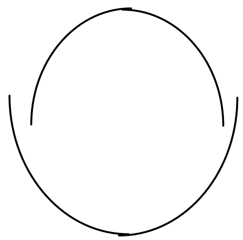
T = 

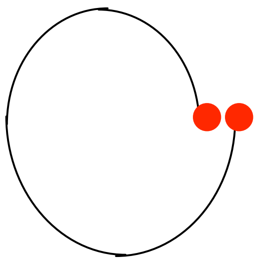
 $\longrightarrow$ compacts $\boldsymbol{K}(L^2(T))$

$$U\pi U^* = \chi_k \pi$$

Ad U to glue

π = regular representation

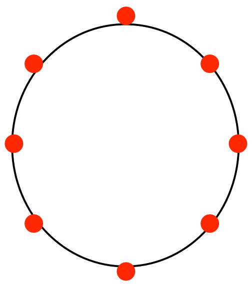
$T =$  $=$ 

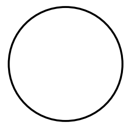
 $\longrightarrow$ compacts $\boldsymbol{K}(L^2(T))$

$U\pi U^* = \chi_k \pi$

Ad U to glue

π = regular representation



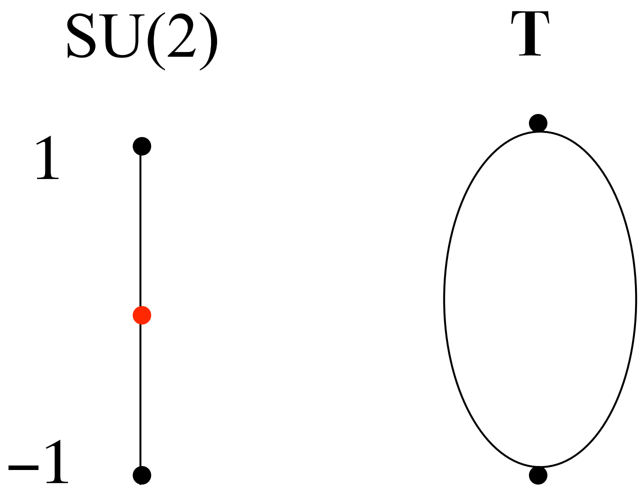
point $\bullet \rightarrow$ 

$K^0_T(\text{point}) \rightarrow {}^kK^1_T(T)$

SU(2) on SU(2)

Conjugacy classes $\mathbf{T}/\mathbf{Z}_2$

$$\begin{pmatrix} t & \\ & t^{-1} \end{pmatrix} \sim \begin{pmatrix} t^{-1} & \\ & t \end{pmatrix}$$

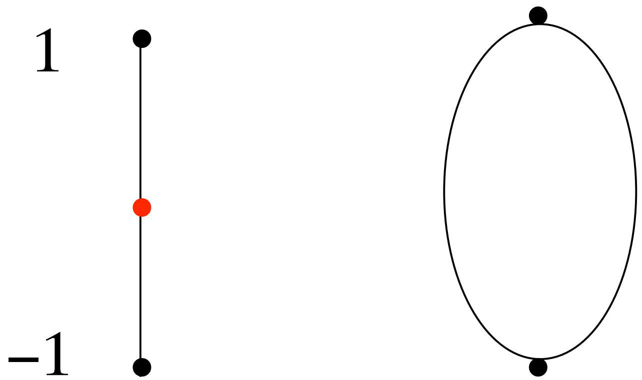


SU(2) on SU(2)

Conjugacy classes $\mathbf{T}/\mathbf{Z}_2$

$$\begin{pmatrix} t & \\ & t^{-1} \end{pmatrix} \sim \begin{pmatrix} t^{-1} & \\ & t \end{pmatrix}$$

SU(2) $\qquad\qquad$ $\mathbf{T}$



$\boldsymbol{K}(L^2(SU(2)))$

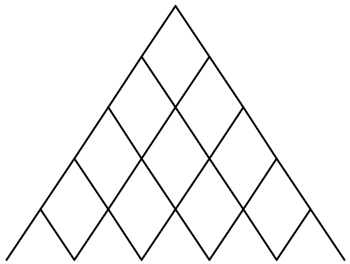
character of stabiliser $\mathbf{T}$

$$U\pi|_{\mathbf{T}}\, U^* = \chi_k\ \pi|_{\mathbf{T}}$$

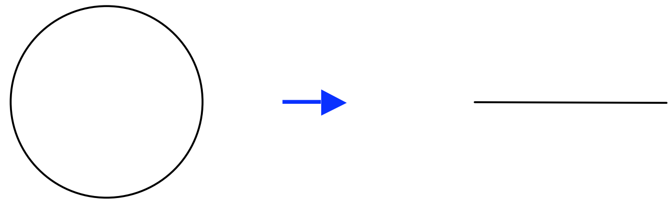
$$H^3_{SU(2)}\, SU(2) \approx \mathbf{Z}$$

$(\otimes_N M_2)^T$

$R_T \approx \Delta$

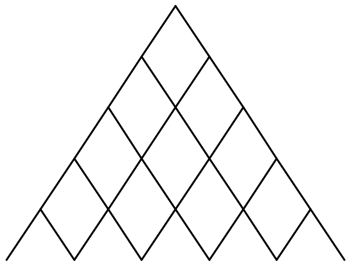


$\omega + \omega^{-1} = z$

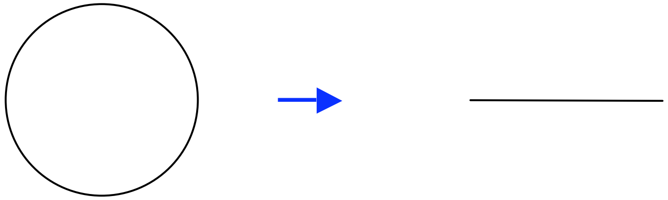


$(\otimes_N M_2)^T$

$R_T \approx \Delta$



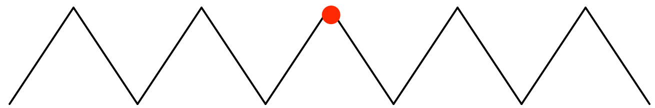
$\omega + \omega^{-1} = z$



$$\dim(\otimes^k M_2)^T = \langle \Delta^{2k}\Omega, \Omega \rangle = \int (\omega + \omega^{-1})^{2k} \, d\omega$$

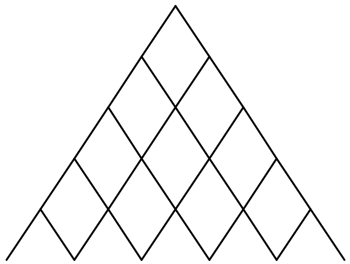
$$= \int \frac{z^{2k} \, dz}{\pi \sqrt{(4 - z^2)}}$$

$\Delta = U + U^*$ on $l^2(Z)$

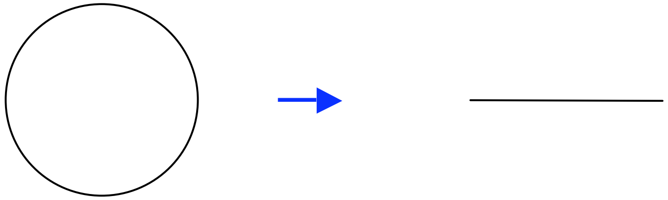


$(\otimes_N M_2)^T$

$R_T \approx \Delta$



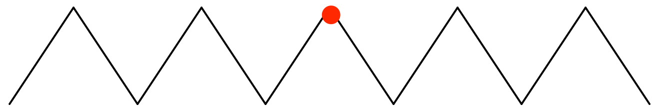
$\omega + \omega^{-1} = z$



$$\dim(\otimes^k M_2)^T = \langle \Delta^{2k}\Omega,\Omega\rangle = \int (\omega + \omega^{-1})^{2k}\, d\omega$$

$$= \int \frac{z^{2k}\, dz}{\pi \sqrt{(4 - z^2)}}$$

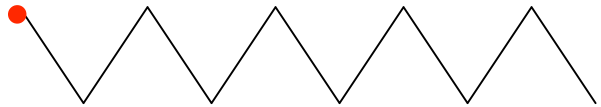
$\Delta = U + U^*$ on $l^2(Z)$

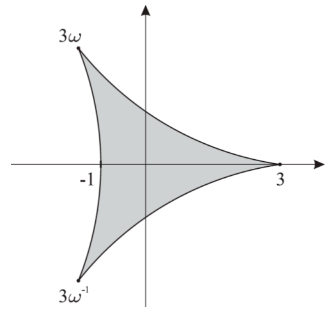


$$\dim(\otimes^k M_2)^{SU(2)} = \langle \Delta^{2k}\Omega,\Omega\rangle =$$

$$\int z^{2k} \frac{\sqrt{(4 - z^2)}}{2\pi}\, dz$$

$\Delta = S + S^*$ on $l^2(N)$



$\mathbf{T}^2 \rightarrow$ 

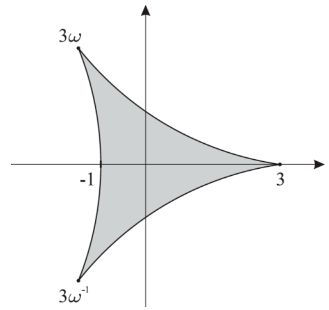
ω_1

ω_2^{-1}

$\omega_1^{-1}\,\omega_2$

$$\rightarrow \quad \omega_1 + \omega_2^{-1} + \omega_1^{-1}\,\omega_2 = z$$

$\mathbf{T}^2 \rightarrow$



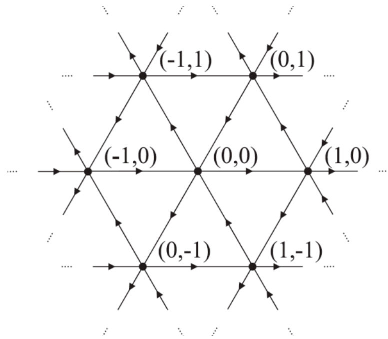
$$\begin{matrix} \omega_1 & & \\ & \omega_2^{-1} & \\ & & \omega_1^{-1}\,\omega_2 \end{matrix} \quad \rightarrow \quad \omega_1 + \omega_2^{-1} + \omega_1^{-1}\,\omega_2 = z$$

$$\Delta = U\otimes 1 + 1\otimes U^* + U^*\otimes U$$

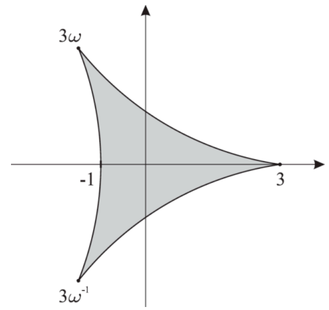
on $l^2(\mathbf{Z}\times\mathbf{Z})$

$$\dim(\otimes^k M_3)^{T_2} = \langle |\Delta|^{2k}\,\Omega,\Omega\rangle$$

$$= \int \frac{3\ |z|^{2k}\ dz}{\pi^2\,(27-18\,|z|^2 + 4z^2 + 4z^{*3} - |z|^4)^{1/2}}$$



$\mathbf{T^2}$ $\rightarrow$

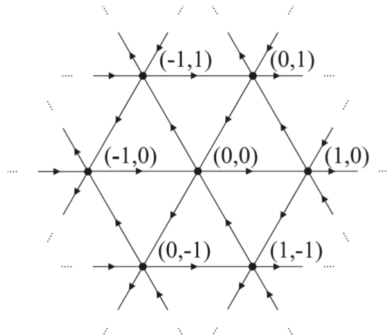


$$\omega_1 \qquad \omega_2^{-1} \qquad \omega_1^{-1}\omega_2 \quad \rightarrow \quad \omega_1 + \omega_2^{-1} + \omega_1^{-1}\omega_2 = z$$

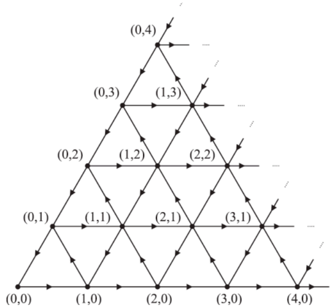
$$\Delta = U\otimes 1 + 1\otimes U^* + U^*\otimes U$$
on $l^2(\mathbf{Z}\times\mathbf{Z})$

$$\dim(\otimes^k M_3)^{T_2} = <|\Delta|^{2k}\,\Omega,\Omega>$$

$$= \int \frac{3\ |z|^{2k}\ dz}{\pi^2\,(27-18\,|z|^2 + 4z^2 + 4z^{*3} - |z|^4)^{1/2}}$$



$$\dim(\otimes^k M_3)^{SU(3)} = <|\Delta|^{2k}\,\Omega,\Omega>$$

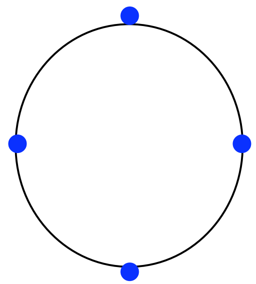


$$= \int \frac{|z|^{2k}\,(27-18\,|z|^2 + 4z^2 + 4z^{*3} - |z|^4)^{1/2}\,dz}{2\pi^2}$$

$$\Delta = S\otimes 1 + 1\otimes S^* + S^*\otimes S \text{ on } l^2(\mathbf{N}\times\mathbf{N})$$

Mod invts finite subgps

T, SU(2)	$\rightarrow$	ADE < 2	affine ADE =2
T^2, SU(3)	$\rightarrow$	$\mathcal{ADE}$ < 3	=3

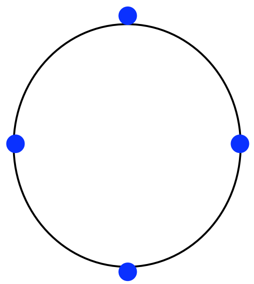


$A^{(1)}_4$

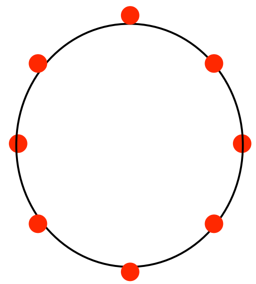
Mod invts finite subgps

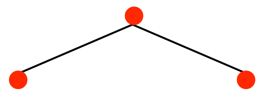
T, SU(2) $\rightarrow$ ADE < 2 affine ADE =2

T^2, SU(3) $\rightarrow$ $\mathscr{ADE}$ < 3 =3



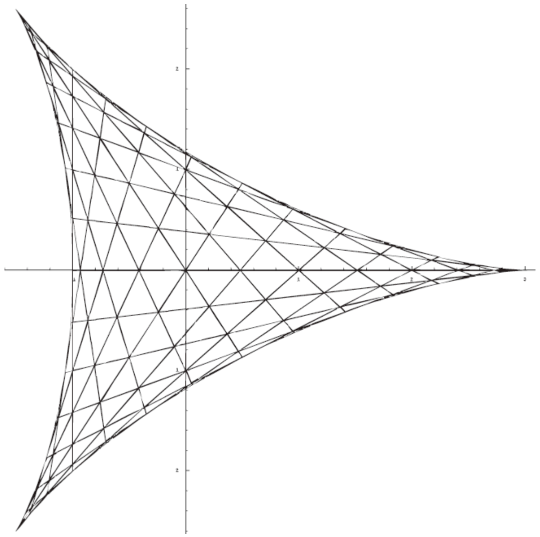
$A^{(1)}{}_4$

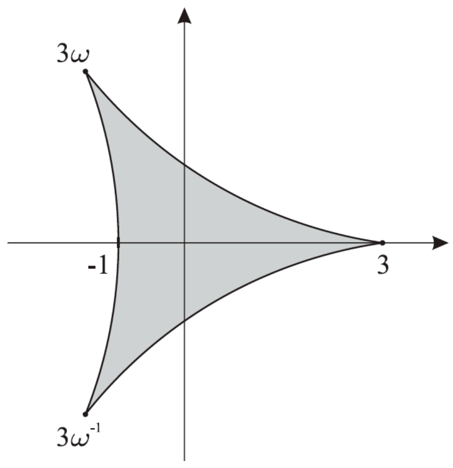




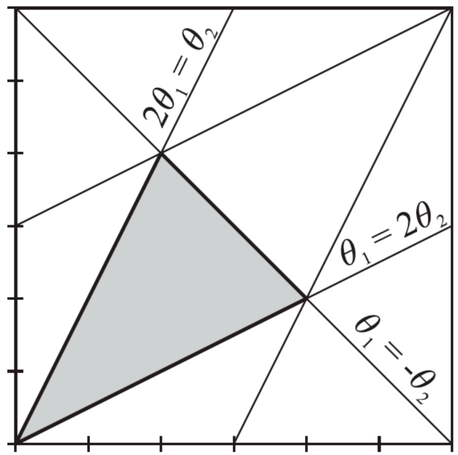
A_3 W × roots of 1

$W = \mathrm{Im}(\omega)^2 = (4-z^2)^{1/2}$

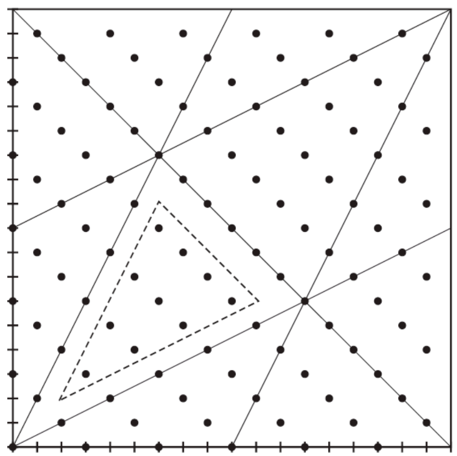


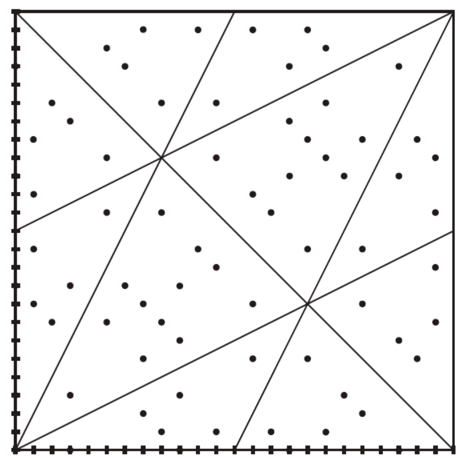


3ω
-1
3
$3\omega^{-1}$



$2\theta_1 = \theta_2$
$\theta_1 = 2\theta_2$
$\theta_1 = -\theta_2$





$SU(3) \rightarrow (E_6)_1$

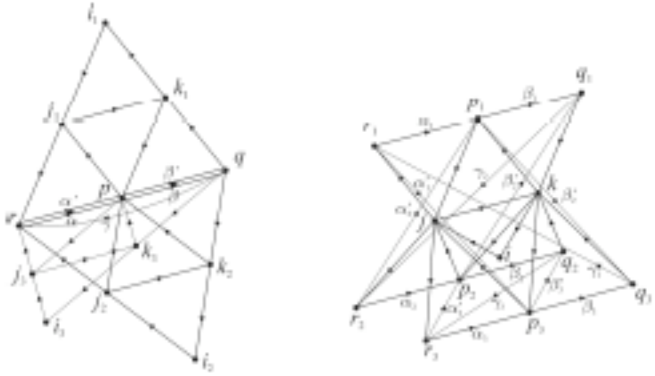


Figure 14: $\mathcal{E}_1^{(12)}$ and $\mathcal{E}_2^{(12)}$

$SU(3) \rightarrow (E_6)_1 \times Z_3$

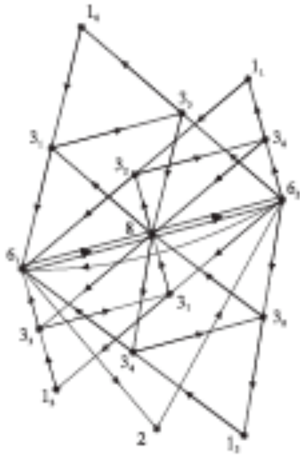


Figure 11: $(F) = \sum(72 \times 3)$

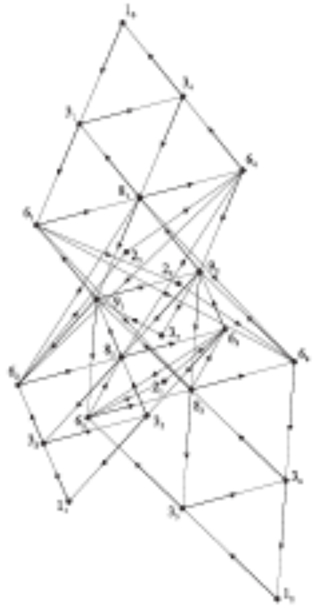
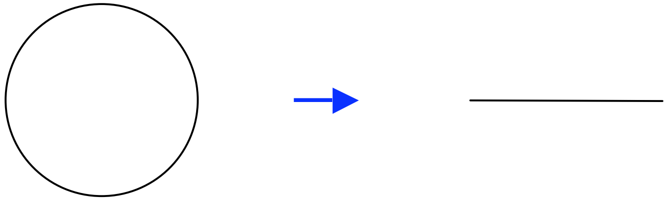


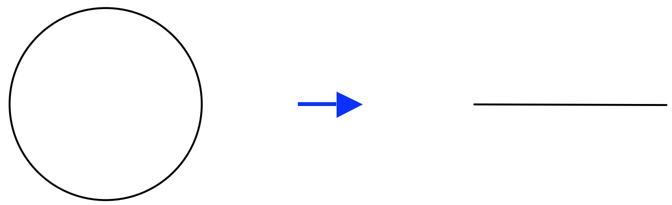
Figure 12: $(G) = \sum(216 \times 3)$



$$\omega \rightarrow \omega + \omega^{-1} = z$$

$$\omega^2 - z\omega + 1 = 0$$

$$\omega = \{z + i(4 - z^2)^{1/2}\}/2 \leftarrow z$$

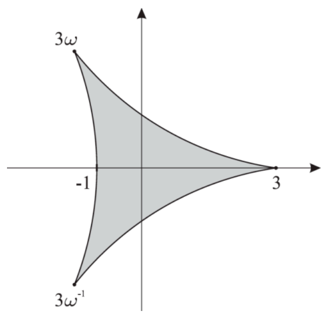


$$\omega \rightarrow \omega + \omega^{-1} = z$$

$$\omega^2 - z\omega + 1 = 0$$

$$\omega = \{z + i(4 - z^2)^{1/2}\}/2 \leftarrow z$$

$\mathbf{T^2} \rightarrow$



$$\omega_1, \omega_2 \rightarrow \omega_1 + \omega_2^{-1} + \omega_1^{-1}\omega_2 = z$$

$$\omega_1^3 - z\omega_1^2 + z^*\omega_1 - 1 = 0$$
$$\omega_2^3 - z^*\omega_2^2 + z\omega_2 - 1 = 0$$

$$\omega = z - P + (z^3 - 3z^*)/3P$$

$$P = \{[27 - 9|z|^2 + 2z^3 + 3.3^{1/2}(27 - 18|z|^2 + 4z^2 + 4z^{*3} - |z|^4)^{1/2}]/2\}^{1/3}$$

$$z \rightarrow \omega, \omega`$$

SU(2) ADE **Cappelli, Itzykson, Zuber**

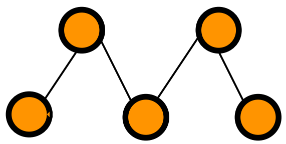
Subfactor realisation: **Ocneanu, Feng Xu, Bockenhauer-Evans-Kawahigashi**

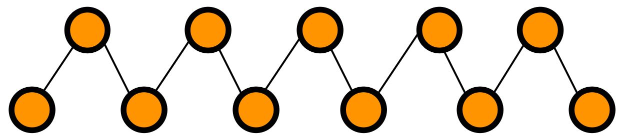
$D_4 \quad |\chi_0 + \chi_4|^2 + 2|\chi_2|^2 \qquad E_6 \quad |\chi_0 + \chi_6|^2 + |\chi_4 + \chi_{10}|^2 + |\chi_3 + \chi_7|^2$

$SU(2)_4 \rightarrow SU(3)_1 \qquad SU(2)_{10} \rightarrow Sp(4)_1$

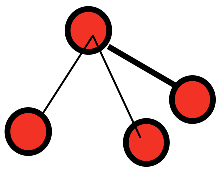
Freed-Hopkins Teleman

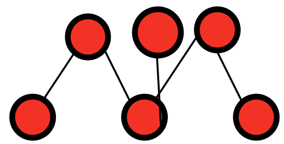
N-N $\quad {}^{\omega}K_G(G)$



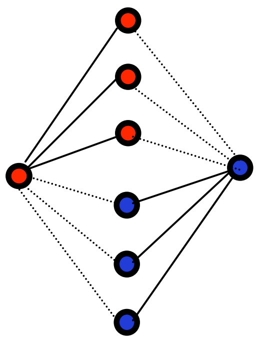


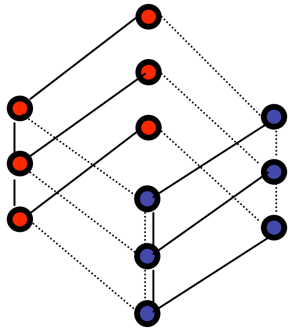
N-M

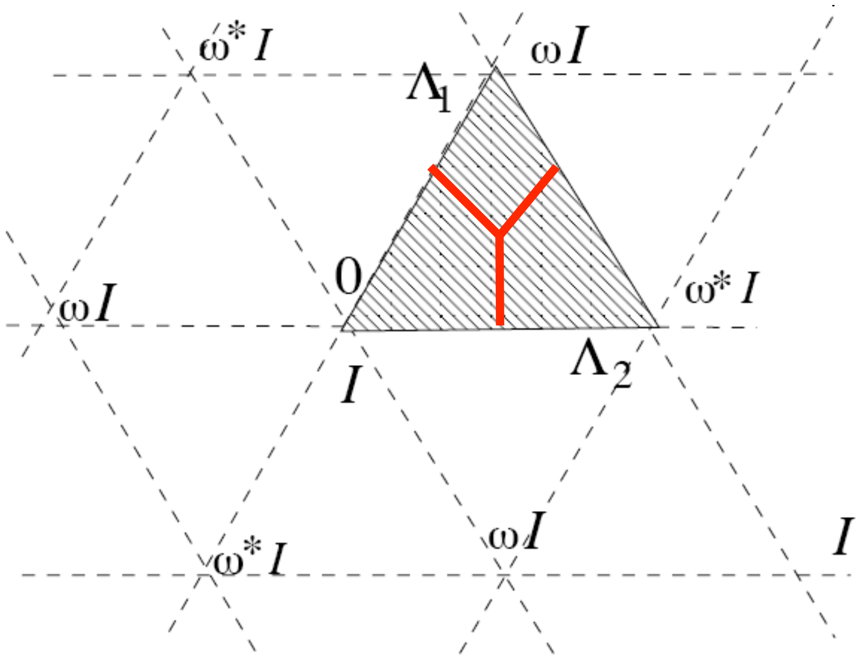


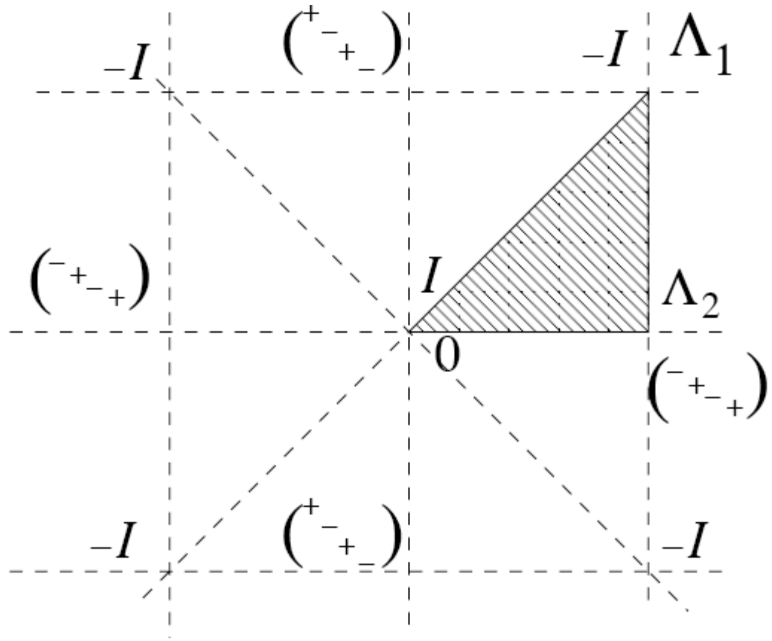


M-M

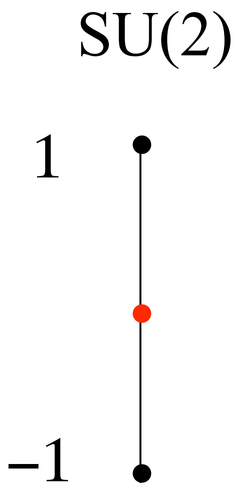


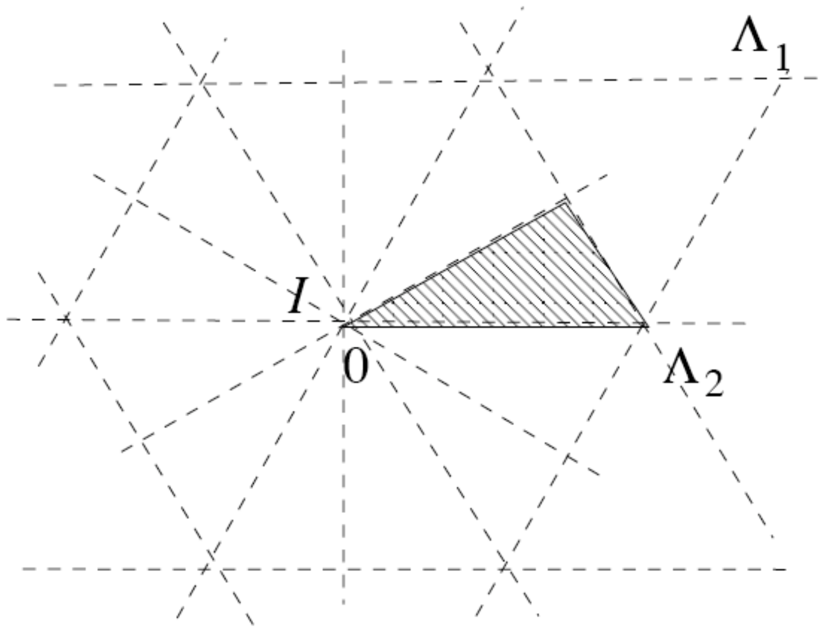


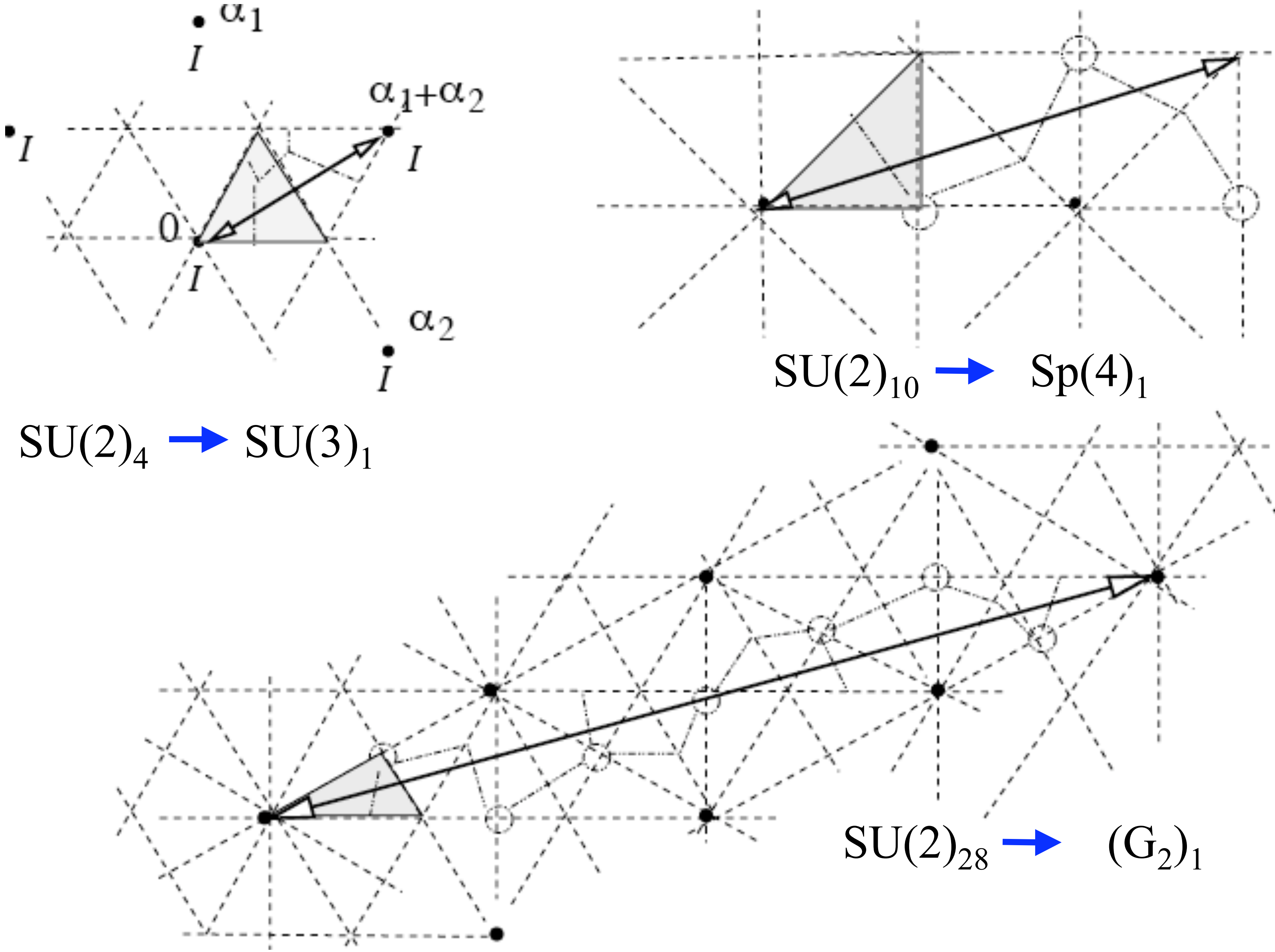

ω* I
ω I
Λ1
0
ω I
ω* I
I
Λ2
ω* I
ω I
I


−I
−I
Λ1
I
Λ2
0
−I
−I

*


SU(2)
1
−1

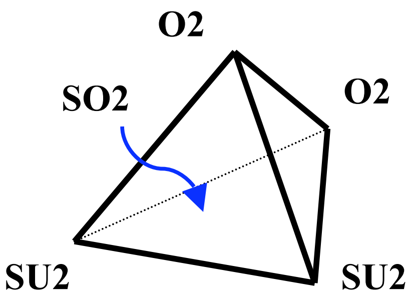

Λ1
I
0
Λ2


α1
I
α1+α2
I
I
0
I
α2
I
SU(2)4 → SU(3)1
SU(2)10 → Sp(4)1
SU(2)28 → (G2)1

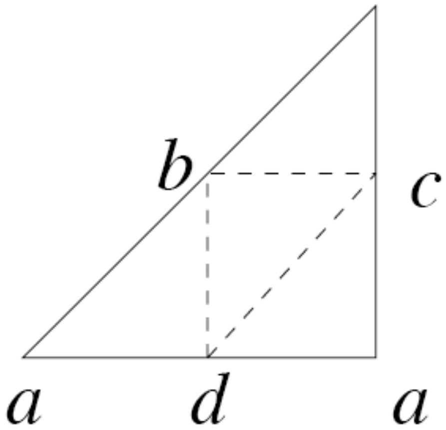
SU(2) on Sp(4)

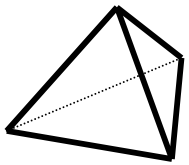
$E^2_{p,q} = \mathrm{Tor}_p^{R(SU2)}(R_{SU2}, {}^{\tau}K^q_{Sp4}(Sp4))$

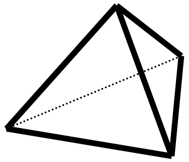
$\longrightarrow$ $K^*_{SU2}(Sp4) = Z^2$

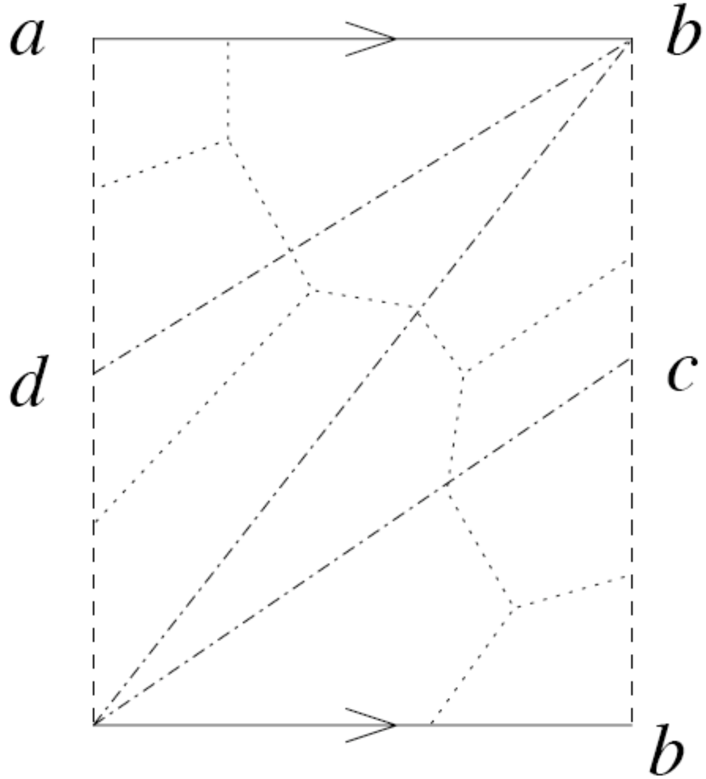


SU(2) on Sp(4)



${}^{\tau}K_0^{SU2}($  $) = 0$

${}^{\tau}K_1^{SU2}($  $) = Z^4$

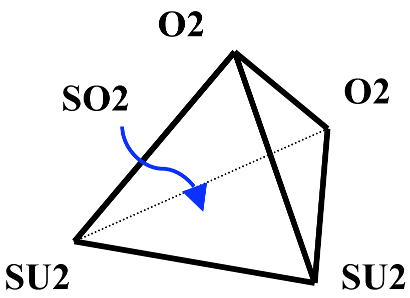


∞-stabilisers
SU(2)
O(2)
SO(2)

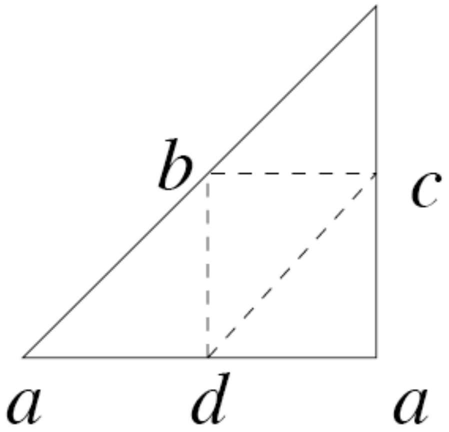
finite stabiliers
E_6
D_5 D_4
A_5 A_3

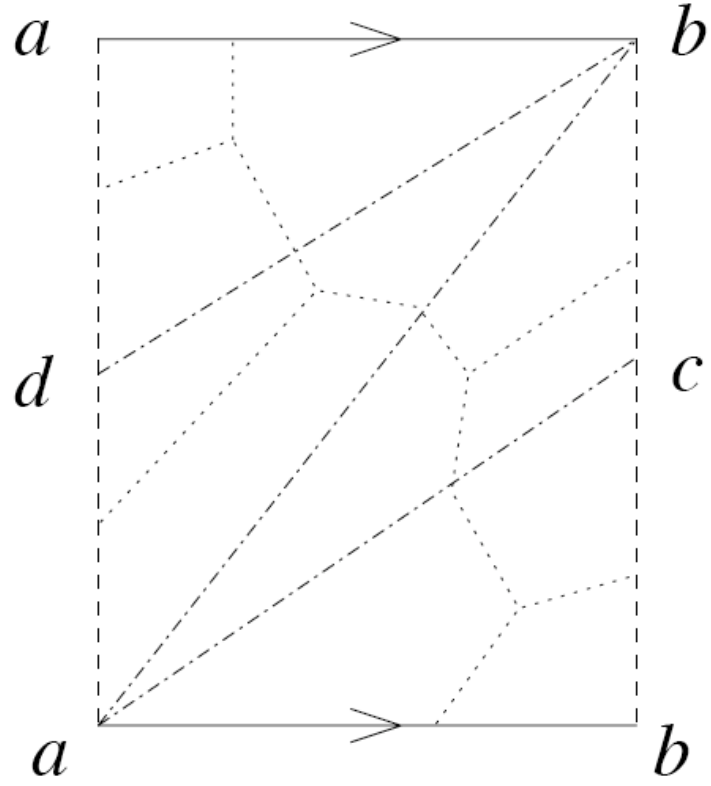
A_1 generic

Evans-Gannon



SU(2) on Sp(4)





finite stabiliers

$\mathbf{E_6}$

$\mathbf{D_5}$ $\mathbf{D_4}$

$\mathbf{A_5}$ $\mathbf{A_3}$

$\mathbf{A_1}$ generic

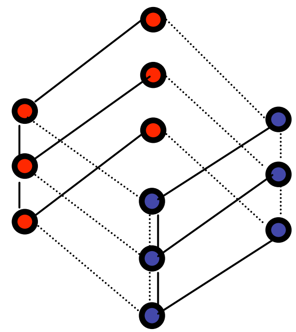
∞-stabilisers

SU(2)

O(2)

SO(2)

$^{\tau}K_0^{SU2}$(non-generics) $= Z^{12}$



Evans-Gannon

$\mathbf{D_4}$ SU(2) → SO(3) → SU(3)

$$\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \rightarrow \quad \mathrm{Re} \begin{pmatrix} \alpha^2 - i\beta^2 & -i\alpha^2 + \beta^2 & 2\xi\alpha\beta \\ i\alpha^2 + \beta^2 & \alpha^2 + i\beta^2 & 2i\xi\alpha\beta \\ -2\xi\alpha^*\beta & -2i\xi\alpha^*\beta & |\alpha|^2 - |\beta|^2 \end{pmatrix}$$

$\xi = e^{i\pi/4}$

$$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & -e^{i\theta} \end{pmatrix} \rightarrow \begin{pmatrix} R_{2\theta} & 0 \\ 0 & 1 \end{pmatrix}$$

$\mathbf{E_6}$ SU(2) → Sp(4)

$$\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \rightarrow \begin{pmatrix} \alpha^3 & 3^{1/2}\alpha^3\beta & \beta^3 & -3^{1/2}\alpha\beta^2 \\ -3^{1/2}\alpha^2\beta^* & (3|\alpha|^2 - 2)\beta & 3^{1/2}\alpha^*\beta^2 & (1-3|\alpha|^2)\beta \\ -\beta^{*3} & 3^{1/2}\alpha^*\beta^{*2} & \alpha^{*3} & 3^{1/2}\alpha^{*2}\beta^* \\ -3^{1/2}\alpha^*\beta^{*2} & (3|\alpha|^2 - 1)\beta^* & -3^{1/2}\alpha^{*2}\beta^* & (3|\alpha|^2 - 2)\beta^* \end{pmatrix}$$

$$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & -e^{i\theta} \end{pmatrix} \rightarrow \quad \text{Diagonal } e^{3i\theta} \;\; e^{i\theta} \;\; e^{-3i\theta} \;\; e^{-i\theta}$$