Dark matter and Wigner's third positive energy representation class

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Abstract

Positive energy representations of the Poincaré group are naturally subdivided into three classes according to their mass and spin content: m>0, m=0 finite helicity and m=0 infinite helicity. For a long time the the quantum field theory of the third class remained a mystery before it became clear that one is confronted with a new kind of "stuff" with very different properties from matter as we know it: unlike normal matter it cannot be localized in compact spacetime regions and its generating quantum fields are semi-infinite spacelike strings. In this note we present arguments that such noncompact stuff is inert apart from gravitational coupling which makes it a perfect candidate for dark matter.

1 String-localization and Wigner's infinite spin representation

Wigner's famous 1939 theory of unitary representations of the Poincaré group $\mathcal{P}$ was the first systematic and successful attempt to classify relativistic particles without relying on a Lagrangian quantization parallelism to classical field theory [1]. As we know nowadays, Wigner's the massive and the massless finite helicity class of positive energy representations of $\mathcal{P}$ cover all known particles and their descriptions in terms of free fields. Coupling these covariant pointlike free fields to form scalar interaction densities is the starting point of perturbation theory on which our present understanding of interacting matter is based.

Whereas the massive representation class $(m > 0, s = \frac{n}{2})$, covers all known massive particles, the massless representations split into two classes which belong to very different unitary representations of the little group. The latter is the invariance group of a lightlike vector i.e. the three-parametric Euclidean group $E(2)$ in two dimensions. Its degenerate representations are the $(m = 0, \pm|h|)$

two-component massless helicity representations, whereas the faithful $E(2)$ representations are infinite dimensional and define the third class of positive energy representations; they were referred to by Wigner as "infinite spin" representations [1].

The localization properties of this third class turned out to be incompatible with pointlocal fields [2]. There are numerous failed attempts which tried to enforce pointlike localization; some of them are mentioned in the first volume of Weinberg's book. Weinberg himself concluded that "nature does not use it". In view of the fact that in those days dark matter was not yet an issue of high energy physics and that all known particles had been identified within Wigner's finite spin/helicity classes, this was a factually correct statement.

All known and conjectured zero mass particles (photons, gravitond, possibly massless neutrinos) belong to the finite helicity class. In formulating interactions of vectormesons in a Hilbert space one has to avoid pointlike vectorpotentials (which inevitably leads to gauge theory in indefinite metric Krein spaces and the presence of ghost operators). The Hilbert space positivity requires to use covariant stringlocal vectormesons [7] but there are still pointlike generated local observables (corresponding to gauge-invariant pointlike fields). The QFT of the third Wigner class is totally different in that *all* (including composite) fields are stringlocal i.e. the localization of the Wigner "stuff" is wholly *noncompact.* We avoid on purpose the terminology "nonlocal" since this stuff admits causal localization in arbitrary narrow spacelike cones; it only cannot be causally localized in compact spacetime regions. To be more precise, in the QFT of ordinary matter there exist useful global observables as conserved charges. But they are always limits of sequences of compact localized operators, whereas the Wigner stuff is "irreducibly" noncompact and defines a completely new type of QFT.

It turns out that the new Hilbert space description of Yang-Mills couplings contains objects whose properties stand in an interesting and fruitful contrast to the Wigner stuff, namely confined gluons and quarks. The interaction of massive gluons and quarks leads to a renormalizable interactions e.g. to massive physical stringlocal gluons which are quite different from gauge-variant pointlike gluon fields. Analogies to the Yennie-Frautschi-Suura [16] treatment of logarihmic on-shell infrared divergencies in QED and their re-summation techniques for leading logarithic divergences in $m \to 0$ suggest that all correlation functions containing in addition to pointlike composites stringlocal vectormesons or quarks[2] vanish; in fact this property should be taken as the definition of confinement. This property scotches the undesired occurance of an acausal process in which colliding compact matter creates noncompact stuff. Whereas gluons and quarks cannot come out of such a process (and therefore do not show up in the energy-momentum balance), the noncompact Wigner stuff, once in this world, cannot change into normal matter, which accounts for its inert behavior.

It is not that the problem of the QFT of Wigners infinite spin representation

[1] In the more recent literature they are sometimes (more appropriately) called "continuous spin" representations.

[2] Apart from $q-\bar{q}$ configurations in which the large distance parts of the two strings cancel and the remaining finite string-bridge is parallel to the spacelike separation of the pair.

class was ignored after Wigner's unsucessful attempts. There were several later equally unsuccessful attempts to press the covariant content of this representation into the form of a pointlike field. The first hint into the right direction was Yngvason's theorem establishing that this representation is incompatible with pointlike localization [2] The group theoretical covariantization method for Wigner's unitary representation theory as used in Weinberg's book did not resolve the problem, and Weinberg's dictum was that nature does not use this representation since matter as we know it can be taken care off in terms of massive or massless finite helicity representations. This was factually correct since the concept of dark matter did not enter the consciousness of particle theorists at that time.

With the help of the more recent intrinsic "modular localization" concept this problem was finally solved in two steps. First the application of this idea to the construction of modular localized subspaces of Wigner's representation space revealed that *all positive energy representations can be localized in arbitrary narrow spacelike cones* (whose cores are semi-infinite spacelike strings) [4]. Apart from the third representation class, this noncompact localization can be sharpened to compact (double cone) regions with pointlike generating wave functions; this is however not possible for the third class since the compact localized subspaces turn out to be trivial. The core of arbitrary small double cones is a point and that of spacelike cones a string. Since the generating covariant wave functions or quantum fields in the first two cases are known to be pointlike, one expected the covariant third class fields to be stringlocal. The associated stringlocal covariant fields were explicitly constructed in [5]. This did not stop the futile attempt to relate the infinite spin Wigner representation with pointlike fields [8] more than 40 years after a No Go theorem to this effect had been established [2] and 7 years after the appropriate stringlocal fields were constructed [5]; an unmistakable sign of increasing fragmation of individual knowledge about particle physics in times of globalization.

The absence of compact localizability for the somewhat mysterious third positive energy representations class has radical physical consequences. Such inert stuff cannot be registered in a counter, neither can it be generated from a collision of ordinary matter. Its main property is its reactive inertness which manifests itself in the absence of almost all properties of ordinary matter, except its coupling to gravity as a consequence of the positive energy property. The arena of action of noncompact matter are galaxies and not earthly laboratories. The astrophysical arguments against identifying massless ordinary matter are not applicable to the noncompact third class stuff. Unlike normal localizable massless matter it cannot escape from galaxies, rather it pervades them and extends into the empty space. Its main and, as will be argued in this paper, only manifestation is the change its presence causes in the galactic gravitational balance.

The identification of the third class Wigner stuff with dark matter has two virtues as compared to the other proposals. It is not an ad hoc invention for explaining dark matter; Wigner's classification is as old as Zwicky's astrophysical observations which led to the dark matter proposal. Furthermore it fulfills the requirement of the possibility of its falsification since any convincing identification of a counter registration event with the astrophysical dark matter disproves the present proposal; the third class Wigner stuff turns out to be inert par excellence, apart from its gravitational coupling.

A lot about physical properties of noncompact matter can be learned by confronting its stringlocal quantum fields with new insights about higher spin ordinary matter [6] [7]. By this we specifically mean the recent discovery of the necessity of using *stringlocal fields in order to maintain the Hilbert space positivity for renormalizable higher spin* $s \geq 1$ *interactions*[3]. The use of $s = 1$ stringlocal fields in Hilbert space (instead of pointlike fields in Krein space) promises to lead to significant progress in the understanding of infrared problems (as e.g. confinement) which arise in the massless limit of interacting physical (Hilbert space) vectormesons. In fact the perception about the relevance of the use of stringlocal vectormesons for maintaining a Hilbert space description arose in the aftermath of the discovery of the field theoretic description of the infinite spin Wigner stuff and the question arises to what extend one may be able to use this connection in the opposite direction.

Massless $s = 1$ interactions are best understood in terms of massless limits of correlation functions of interacting massive stringlocal vectormesons. This is because QFTs with a mass gap have a rather simple relation between fields and particles which manifests itself in the validity of time dependent (LSZ) scattering theory and the ensuing Wigner-Fock structure of the Hilbert space. These properties become blurred in the massless limit and lead to perturbative infrared divergencies whose physical consequences can be investigated by YFS resummation techniques of leading infrared divergencies [16]. The analog for the Wigner stuff would be to represent its stringlike fields as a massless limit of a massive high spin representation in which the vanishing of the decreasing mass is coupled to a growing spin. Unfortunately it is presently not known how to do that or even if this is possible.

Falling short of such an ambitious goal, the main point of the present work is to use what has been learned from the Hilbert space reformulation of the BRST gauge theory for a better understanding of possible physical manifestations (or rather their absence, apart from the gravitational coupling) of the Wigner stuff as a consequence of its known very strong semi-infinite string-localization. A placative formulation of the result would consist in viewing darkness and confinement as the two opposite flanks of normal matter: confinement is connected with a property of particular interacting stringlocal fields which diappears in the $m \to 0$ limit of stringlocal massive vectormesons, whereas dark matter, once in this world, cannot disappear by being converted into normal matter[4]. The foundational reasons, namely the havoc with the causality principle which a change of compact spacetime localized matter into irreducible noncompact stuff would

[3] The standard setting for renormalizable $s = 1$ interactions is the pointlike Becchi-Rouet-Stora-Tuytin (BRST) formulation of operator gauge theory which replaces Hilbert space by an indefinite metric Krein space.

[4] Whereas Inert matter carries energy-momentum, confined fields/particles i.e. fields which vanish in the massless limit, do not contribute to the energy-momentum balance.

cause, are shared but the physical consequences are very different.

Some details on the connection between Wigner's positive energy representations with localization can be found in the next two sections. The last section comments on the use of the inert Wigner stuff as a possible explanation behind the observed astrophysical darkness.

2 Point- and string-like generating fields of positive energy representations

Recent progress on foundational localization problems revealed that the generating fields of the infinite spin Wigner stuff representation class is stringlocal. We will only present the results and refer the reader for their derivation to [5] [9]. For the selfconjugate bosonic case (to which we limit our exemplary presentation) they are of the form

$$\Psi(x,e) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \{e^{ipx} u^\alpha(p,e) \cdot a^*(p) + e^{-ipx} \overline{u^\alpha(p,e)} \cdot a(p) + \} \frac{d^3p}{2\omega} \tag{1}$$

$$u \cdot \bar{u} := \int d^2k \delta(k^2 - \kappa^2) u(k) \bar{u}(k), \quad u^\alpha(p,e)(k) = e^{-i\pi\alpha/2} \int d^2z e^{ikz} (B_p \xi(z) \cdot e)^\alpha,$$

$$\xi(z) = (\frac{|z|^2 + 1}{2}, z_1, -z_2, \frac{|z|^2 - 1}{2}), \quad e^\mu e_\mu = -1$$

$$(D^\kappa(c, R(\theta))\varphi)(k) = e^{ic\cdot k} \varphi(R^{-1}(\theta)k), \; \mathfrak{h} = \; L^2(\mathbb{R}^2, \delta(k^2 - \kappa^2) d^2k)$$

Here the intertwiners $u^\alpha(p,e)(k)$ are p,e -dependent functions on the two-dimensional k-space on which the 2-dim Euclidean group $E(2)$ acts (c translation, θ rotation) and in this way defines a representation on a Hilbert space $\mathfrak{h}$. The Pauli-Lubanski invariant κ is a continuous parameter ("the continuous spin/helicity") which characterizes the $E(2)$ representation and defines a Casimir invariant of the positive energy representation of $\mathcal{P}$ associated to the stuff. The u are called "intertwiners" because they convert the unitary Wigner representation acting as the adjoint representation on $a^{\#}(p)(k)$ into the covariant transformation of the $\Psi(x,e)$. They also lead to stringlike causality i.e. the vanishing of the commutator for spacelike separations of the two strings[5] in relative spacelike positions

$$U(\Lambda,a)\Psi(x,e)U(\Lambda,a)^* = \Psi(\Lambda x + a, \Lambda e) \tag{2}$$
$$[\Psi(x,e), \Psi(x',e')] = 0, \; for \; x + \mathbb{R}_+ e >< x' + \mathbb{R}_+ e'$$

The derivation of the formula for the covariant intertwiners uses ideas from modular localization [5] in addition to the group theoretic properties which were already used for the construction of the quantum fields for the massive and finite helicity massless class by Weinberg [3].

[5] We only need the simplest realization of Wigner strings (bosonic, self-conjugate, without additional spinorial indices). Covariant strings are necessarily straight.

The transcendental u-intertwiners lead to rather involved two-point functions and propagators

$$\langle\Psi(x,e)\Psi(x',e')\rangle = \frac{1}{(2\pi)^3}\int e^{-ip(x-x')}M(p;e,e') \tag{3}$$

$$M(p;e,e') = \int u(p,e)(k)\overline{u(p,e')}(k)\delta(k^2-\kappa^2)d^2k\frac{d^3p}{2p_0} \tag{4}$$

$$\langle\Psi(x,e)\Psi(x',e')\rangle \rightarrow \langle T\Psi(x,e)\Psi(x',e')\rangle \quad by \quad \frac{d^3p}{2\left|\vec{p}\right|} \rightarrow \frac{1}{2\pi}\frac{1}{p^2-i\varepsilon}d^4p$$

where the third line denotes the transition from the two-pointfunction to the propagator by changing the p-integration.

For later use we also present the corresponding representations for pointlocal fields from the massive and finite helicity Wigner class (b refers to antiparticles)

$$\psi^{A,\dot{B}}(x) = \frac{1}{(2\pi)^{3/2}}\int(e^{ipx}u^{A,\dot{B}}(p)\cdot a^*(p) + e^{-ipx}v^{A,\dot{B}}(p)\cdot b(p))\frac{d^3p}{2p_0} \tag{5}$$

The intertwiners $u(p)$ for $m>0$ are rectangular $(2A+1)(2B+1) >< (2s+1)$ matrices which intertwine between the unitary $(2s+1)$-component Wigner representation and the covariant $(A.\dot{B})$ spinorial representation; the a,b refer to particle and antiparticle creation/annihilation operators. For the $m=0$ representations the formula is the same, except that dot stands for the inner product in a two-dimensional space (the space of the two helicities $\pm\left|h\right|$). Another difference between the massive and the massless case is the range of possible spinorial indices; for a given physical spin s the range of spinorial (half)integer spinorial representation indices of the homogeneous Lorentz group is restricted by

$$\left|A-\dot{B}\right| \leq s \leq A+\dot{B}, \quad m>0 \tag{6}$$

$$\left|A-\dot{B}\right| = \left|h\right|, \quad m=0 \tag{7}$$

the second formula shows that the the vector representation $A=1/2=B$ does not occur for $m=0$ i.e. *pointlike massless covariant vectorpotential are not consistent with the Hilbert space positivity of quantum theory* (the mentioned clash of massless $s\geq1$ pointlike tensorpotentials with Hilbert space positivity).

As mentioned, the discovery of the stringlocal noncompact third class Wigner stuff was the beginning of a systematic study of *stringlocal* fields for the two pointlike generated finite spin/helicity representation classes associated with compact localizable ordinary matter. The absence of pointlike massless vectorpotentials (more generally $s\geq1$ tensor potentials) led to the definition of covariant stringlocal vectorpotentials with the covariance and locality property

(2)

$$A_\mu(x.e) := \int_0^\infty e^\nu F_{\mu\nu}(x+se)ds, \quad e^\mu e_\mu = -1 \tag{8}$$
$$\langle A_\mu(x,e)A_\nu(x',e')\rangle = \frac{1}{(2\pi)^{3/2}}\int e^{-ip(x-x')}M_{\mu\nu}(p;e.e')\frac{d^3p}{2p_0}$$
$$M_{\mu\nu}(p;e.e') = -g_{\mu\nu} - \frac{p_\mu p_\nu e\cdot e'}{(p\cdot e - i\varepsilon)(p\cdot e' + i\varepsilon)} + \frac{p_\mu e_\nu}{p\cdot e - i\varepsilon} + \frac{p_\nu e'_\mu}{p\cdot e' + i\varepsilon}$$

where the ε-prescrition defines the two-point distributions as boundary values of analytic functions. Different from the pointlike Proca potential, these stringlocal fields permit a massless limit [6] [7].

There is no problem with the existence of pointlike $s \geq 1$ higher spin potentials, apart from the fact that their short distance dimension increases with spin $d_{sd} = s+1$. Already for $s=1$ the $d_{sd} = 2$ pointlike Proca vectorpotential $A_\mu^P(x)$ is too singular in order to permit a renormalizable interaction density within the power-counting limit $d_{sd}^{int} \leq 4$. But the massive stringlocal vectorpotential constructed according to (8) has $d_{sd} = 1$, in fact *all* $d_{sd} = s+1$ pointlike tensor potentials have stringlocal siblings of $d_{sd} = 1$. The field-fluctuations in the directional spacetime variable e has led to a reduction of the strength of x-fluctuations from $s+1$ to 1.

For $s=1$ and $m>0$ there exists also a $d=1$ stringlike scalar

$$\phi(x,e) = \int_0^\infty e^\mu A_\mu^P(x+se)ds \tag{9}$$
$$F_{\mu\nu} := \partial_\mu A_\nu^P - \partial_\nu A_\mu^P, \quad A_\mu(x,e) := \int_0^\infty e^\nu F_{\mu\nu}(x+se)ds \tag{10}$$

The second line presents the stringlocal field in terms of its pointlike Proca sibling so that its two.pointfunction (8) follows from that of the Proca field; all other two-pointfunctions, including the mixed A-ϕ ones, can be derived via (9) from that of the Proca field. In fact the 3 fieds obey the linear relation

$$A_\mu(x,e) = A_\mu^p(x) + \partial_\mu\phi(x,e) \tag{11}$$

in which the explicit dependence of line integrals has been absorbed into the defintion of stringlocal fields. The relation of $d_{sd} = 1$ stringlocal tensorpotentials of spin s to their pointlike siblings requires the presence of s lower spin $\phi's$.

It turns out that the stringlocal scalar ϕ plays an important role in the formulation of stringlike interactions; it "escorts" the $A_\mu(x,e)$ potential and enters explicitly the interaction densities, although it does not add new degrees of freedom[6]. All three fields are linear combinations of the three $s=1$ Wigner creation/annihilation operators $a^\#(p,s_3)$ with different u-intertwiners; in fact

[6]It plays an important role in the coupling of a massive vectormeson to a Hermitian H-field which turns out to be the correct formulation (no symmetry-breaking, no spontaneous creation of vectormeson masses) of the Higgs model [6].

(11) is nothing else than a linear relation between three intertwiners. Pointlike intertwiners are matrices with *polynomial* entries in p of degree $d_{sd} = s$ whereas the p-e dependence of stringlocal tensor potentials and their lower spin $\phi's$ is *rational* in p, e. Such stringlocal free fields are "reducible" in the sense that they can be written as semi-infinite line integrals over pointlike observables. This is to be compared to the transcendental p-e dependence of the third class intertwiners (1) which represents *irreducible* strings whose smearing in x, e results in noncompact (referring to spacetime localization) stuff. Nonlocal objects as e.g. conserved global charges also appear in normal compact localizable QFTs, but they can always be described as global limits of local operators, except for the Wigner stuff where such a local approximation is not possible.

This picture about free fields and their associated particles changes drastically in the presence of interactions. Whereas interactions involving massive $d_{sd} = s+1$ pointlike fields are nonrenormalizable, the transformation of such interactions into their stringlike counterpart permits to construct renormalizable interactions for any spin. The stringlike renormalizable formulation shows that *behind the pointlike failure of renormalizability is a weakening of localization in the sense of nonexistence of pointlike Wightman fields whose role is taken over by renormalizable stringlocal fields*; in short, the breakdown of pointlike renormalizability is caused by the weakening of localizability from compact to noncompact localization. This is interesting because it shows that the weakening of localizability is interwoven with a radical worsening of pointlike short distance behavior which manifests itself in a breakdown of renormalizability. Wightman localizability (operator-valued Schwartz distributions) amounts to polynomial boundedness in momentum space; this can only be restored by the reformulation in terms of stringlike fields.

It turns out that behind the point- versus string-like localization is the powerful Hilbert space positivity: *the stringlike localization is the tightest localization which is consistent with Hilbert space positivity* i.e. for generating the net of localized algebras which is the algebraic description of QFT [10] one does not need generating fields which are localized e.g. on spacelike hypersurfaces. Here pointlike is viewed as a special case of stringlike (i.e. pointlike $\simeq$independence on e). Renormalization theory shows that pointlike renormalizability and the absence of massless infrared divergencies is limited to $s < 1$, whereas renormalizability if interactions involving $s \geq 1$ fields requires string-localization and may lead (depending on the interaction) to infrared divergencies in the massless limit [6]. In other words the origin of infrared divergencies (which only start to appear for $s \geq 1$ interactiong fields[7]) is the long range interaction caused by massless stringlike fields. Pointlike interactions for $s < 1$ are compatible with the Hillbert space and do not cause infrared divergencies.

At this point the attentive reader may want to know how quantum gauge theory fits into this new setting. Quantum gauge theory abandons the Hilbert space positivity and keeps instead the pointlike formalism. Whereas the classical

[7] Couplings of $s < 1$ fields with interactions within the power-counting limitation have well-defined infrared-divergence free massless limits.

gauge formalism fits well into classical field theory, its quantum counterpart violates the Hilbert space positivity (which is the Holy Grail of quantum theory). Though this property is later recovered for the gauge-invariant fields whose application to the vacuum create a smaller Hilbert space (the vacuum sector), the formalism does not provide physical operators whose application to the vacuum describe charged states; in fact the unphysical nature of gauge-dependent allegedly charge-carrying matter fields is evident from the observation that the Maxwell charge of the associated states (which they create from the vacuum) vanishes [13]. The 70 year use of quantum gauge theory, which entered QFT through Lagrangian quantization, and the discovery of successful recipes to navigate around these shortcomings (viz. the photon-inclusive cross sections in QED, the prescriptions of physical hadrons in terms of composites of gauge-dependent quarks) led to the loss of awareness about its limitations. From a mathematical viewpoint perturbation theory in an indefinite metric (Krein space) setting is pure combinatorics outside the range of functional analytic or operator algebraic control (violation of the Cauchy-Schwarz inequality,,..).

The new stringlocal setting maintains the powerful positivity restriction coming with the Hilbert space and with it the applicability of operator algebraic methods at a seemingly small price of weakening of localization from point- to string-like. It opens the path to the construction of stringlocal physical matter- and Yang-Mills fields and establishes the conceptual prerequisites for studying the remaining important open problems of QFT including the unsolved problem of confinement. In the present work QFT is always meant in a Hilbert space setting unless the Krein space gauge theory setting is explicitly mentioned.

The reason for recalling recent results about the use of stringlocal fields (which are presently revolutionizing our ideas about localization properties and their physical manifestations of normal matter [6] [7]), is that the best characterization of the infinite spin Wigner stuff is in terms of the *absence* of most properties of matter as we know it. This problem will be taken up in the next section.

3 Wigner's infinite spin stuff and matter as we think we know it

Since causal localization is the foundational property of QFT, the main task of particle physics is to explain the wealth of observed properties as different manifestations of this unifying principle in the context of different models of QFT. It requires in particular to understand in which sense confinement in QCD and spacetime properties of infraparticles in QED are related to localization. The Hilbert space setting suggest a clear picture behind such infrared problems.

Its starting point is the mass gap property which secures the Wigner-Fock particle structure of the Hilbert space. The resulting field-particle relation is described by a structural theorem which states that in a theory with local observables (which define the vacuum sector) the superselection-charge carrying in/out scattering states can be described in terms of time-dependent LSZ scattering theory applied to operators localized in arbitrary narrow spacelike cones (whose cores are strings) [11]. In this case the new perturbation theory based on stringlocal fields (SLF) permits the construction of *singular* polynomially unbounded pointlike fields whose direct perturbative use would have led to nonrenormalizability [6].

This changes abruptly in the limit of massless vectormesons. In case of QED the use of the Gauss theorem (appropriately adapted to QFT [12]) shows that the asymptotic direction of the spacelike cone-localization of charge-carrying operators can not be changed by unitary operators; in other words the directions e of the generating semi-infinite stringlocal fields are "rigid". In particular the Lorentz covariance outside the vacuum sector is spontaneously broken [13] and the culprits are soft photon clouds which hover along the spacelike semi-line $x + \mathbb{R}_+ e$.

This accounts for a change in the field-particle relation; in particular the mass-shell of the charged particle which in theories with mass-gap leads to a $\theta(p_0)\delta(p^2 - m^2)$ contribution (or a $(p^2 + m^2)^{-1}$ pole contribution in the time-ordered functions) "dissolves" into the continuum in form of a milder cut singularity with threshold starting on the mass-shell. In this case a spacetime dependent collision theory of infraparticles which generalizes the LSZ scattering theory[8] does not yet exist and one has to take recourse to the well-known successful *prescription* in terms of photon-inclusive cross-sections [16]. The perturbative manifestations are the well-known on-shell logarithmic infrared divergencies whose resummation in leading orders lead to power behavior in the infrared cutoffs. The removal of the infrared regularization implies the vanishing of scattering amplitudes which can be avoided by passing to the photon-inclusive cross-sections before removing the infrared cutoff [16]. The new stringlocal setting promises to lead to a spacetime understanding of these prescriptions in which the ad hoc noncovariant infrared regulator is replaced by the more natural mass of the stringlocal vectormeson while the unphysical (gauge-dependent) pointlike matter fields pass to stringlocal charge-carrying fields in Hilbert space.

As mentioned above the situation changes in the limit of massless gluons. There remains a significant difference between QED and QCD; in QED the vectorpotential can be written as a semi-infinite line integral over a pointlocal observable (the field strength), whereas stringlocal interacting gluon fields cannot be written in this way. In the latter case the strings are "irreducible". In the massless limit the singular pointlike siblings of the stringlocal fields disappear and the interacting massless gluon matter becames inherently noncompact, whereas certain e-independent composites generate (corresponding to the gauge-invariant observables of gauge theory) the compact localizable part of Y-M matter. If such intrinsic noncompact matter could emerge from a collision of compact matter one would have serious problems with causality. This suggests to define gluon/quark confinement of particle theory as the *vanishing of corre-*

[8] The application of LSZ would lead to vanishing large time asymptotic limits since the weaker threshold singularity cannot compensate the dissipation of wave packets.

lation functions which contain besides composite pointlike observable fields also *stringlocal gluon and quark fields*[9]

The attractive aspect of this theoretical definition of the physical meaning of confinement is that it does not only explain the observational situation but it can in principle also be checked in terms of an extension of the Yennie-Frautschi-Suura infrared resummation techniques applied to the off-shell logarithmic infrared divergencies of stringlike gluon correlations in the massless limit $m \rightarrow 0$, using the vectormeson mass as a natural infrared regularization parameter. In the pointlike BRST gauge setting there are no physical massive gluon fields which one could use in such a calculation. One expects that the Hilbert space positivity plays an essential role in the understanding of physical aspects of infrared phenomena. It is interesting to note that, whereas this confinement mechanism would perturbative accessible by resummation of leading logarithmic infrared divergencies in the $m \rightarrow 0$ limit, the problem of bound states (gluonium, hadrons as bound states of quarks) associated with pointlike composite fields remains still outside the range of presently known perturbative resummation methods.

An almost trivial illustration of a spectrum changing mechanism is provided by exponentials of a massive free field $\phi(x)$ in two spacetime dimensions whose two-pointfunction is logarithmically divergent in the massless limit $m \rightarrow 0$; so that the perturbative expansion of correlations of the exponential fields $exp \pm ig\phi$ lead to logarithmically infrared divergent series in g. On the other hand the exact limiting behavior is a power law. Muliplying the exponential field with a power in the mass $m^{\alpha} expig\phi$, the a can be adjusted in such a way that all expectation values of the exponential field and its Hermitian conjugate remain finite in the massless limit. The result is the emergence of the charge conservation law: all correlations for which the $+g$ charges do not compensate the $-g$ charges vanish, so that only neutral correlations remain in the massless limit. The perturbative mechanism for the infrared divergencies of the massive gluon-quark system in the limit of vanishing gluon mass is expected to be analogous; but since the gluons are chargeless and the e-stringlocal nature has to be taken into account, the expected analogous results is that only correlations of pointlike composites and $q - \bar{q}$ pairs with a finite connecting string (the charge congugation inverts the e-direction) remain.

If the model of QCD is really capable to describe confinement, there is no alternative to this picture about implications of perturbative logarithmic infrared divergencies. The necessary perturbative resummation techniques should be similar to those used by Yenni-Frautschi-Suura [16] to show that the scattering amplitudes (but not the off-shell correlation functions) of charged particicles and a finite number of photons vanish in the limit of vanishing photon mass. In all these $m \rightarrow 0$ limits it is important that the interpolating massive theory fulfills the Hilbert space positivity which requires the use of stringlocal fields and excludes the BRST gauge setting whose physical range of validity does not

[9] The only exception are $q - \bar{q}$ pairs in which the e-directions are parallel to the spacelike separation of the end points so that the strings compensates apart from a piece which connects the endpoints ("bridged" $q - \bar{q}$ pairs)..

extend beyond the gauge invariant observables.

The stringlocal free fields of the Wigner stuff are irreducible in a very strong sense. Whereas in the gluon/quark model the irreducibility referred to the model-defining interacting strings whose confinement is manifested in the vanishing of correlation functions containing gluon fields or "unbridged" $q-\bar{q}$ pairs (see above), the absence of causality violation changes of the Wigner stuff into normal matter explain its inert behavior. This should again be seen in form of infrared divergencies in perturbative calculations of interactions the stringlocal stuff.

Indeed, the attempt to use the transcendental propagators (3) of fields associated with the Wigner stuff leads to severe perturbative infrared divergencies. In this case there exists no massive model in Hilbert space whose $m \to 0$ limit describes the infinite spin representations and could be used as a natural covariant infrared regularization[10]. The inert Wigner stuff and confinement share the string-localization of their fields; but in the Wigner case it is a property of a free field which by causality is prevented from entering an interaction which transforms it into compact matter, whereas for QCD the interacting gluons and quarks are confined i.e. they cannot escape except in the form of pointlocal composites.

The free QFTs which are canonically associated with positive energy representations of the Poincaré group fulfill both causality requirements of the foundational causal localization principle namely the spacelike Einstein causality and the timelike *causal completeness property* [14]. For models which permit a formal representation in terms of Lagrangian quantization this is a consequence of the hyperbolic character of the propagation of solutions of Euler-Lagrange equation, but the Wigner stuff does not permit such a representation. In the algebraic setting of local quantum physics *causal completeness* is the equality of the *outer approximation* of an $\mathcal{O}$-localized algebra [11] in terms of wedge-localized algebras $\mathcal{A}(W)$ is equal to inner approximation in terms of double-cone (diamond)-localized algebras $\mathcal{A}(\mathcal{D})$

$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'') \; where \; \mathcal{A}(\mathcal{O}) = \cup_{\mathcal{D}\subset\mathcal{O}}\mathcal{A}(\mathcal{D}), \;\; \mathcal{A}(\mathcal{O}'') = \cap_{W\supset\mathcal{O}}\mathcal{A}(W)$$

For the noncompact Wigner stuff $\mathcal{O}$ is a noncompact convex region which extends to spacelike infinity and instead of D the inner approximation is effected in terms of spacelike cones $\mathcal{C}$.

In passing it is interesting to note that this important causal completion property is violated in certain cases which appeared in the literature without the protagonists of these proposals having noticed this deficiency. It is always violated on one side in the mathematical AdS-CFT isomorphism (i.e. it cannot hold simultaneously on both sides) and in proposals about extra dimension and attempts to use Kaluza-Klein dimensional reductions in QFT (outside quasi-classical approximations) [14] [15].

[10] A representation of the stuff as a limit of ordinary matter would require a decreasing mass accompanied by an increasing spin; it is not known whether such a representation is possible.

[11] The algebra generated by smearing fields with spacetime $\mathcal{O}$-supported testfunctions; $\mathcal{O}'$ denotes the causal complement of $\mathcal{O}'$ and $\mathcal{O}''$ is its causal completion.

Since causal localization is the defining principle of QFT it would be surprising if nature misses the chance to realize its foundational principle in the guise of Wigners continuous helicity class. As a result of its noncompact spacetime localization and its reactive inertness apart from gravity, its only possible arena of physical manifestation would be galaxies and not earthly laboratories. There is presently no known theoretical physical principle which forbids nature to manifest itself as dark (inert apart from gravitation) matter.

There remains the question of how such inert noncompact matter can be made compatible with its formation in a big bang. In models in which dark matter is identified with (known or still to be discovered) forms of ordinary (compact localized) matter this perfecy inertness cannot be realized. Such explanations have potential problems with astrophysical observations of upper limits which may be too low in order to account for the gravitationally inferred dark matter content whereas for purely gravitationally coupled matter there are no limitations coming from the "visible" forms of matter.

4 Gravitational coupling, concluding remarks

As all positive energy matter, the Wigner stuff couples to the gravitational field. The rough argument uses the effective mass obtained from the Einstein relation $E = mc^2$. It also possesses a conserved stringlocal energy-momentum tensor (use the wave equation for Ψ)

$$T^{\mu\nu}(x,e) = \ :\partial^{\mu}\Psi(x,e)\partial^{\nu}(x,e): \tag{12}$$

whose expectation in suitable states can be used on the right hand side of the Einstein-Hilbert field equation[12]. It would be interesting to consider expectation values in quasifree states to study the induced gravitation of noncompact matter. Note that the noncompact Wigner stuff does not permit a pointlike energy-momentum tensor, the stringlike representation (12) is the tightest possible local representation.

It is clear that the only possible physical use of this stuff is as a candidate for dark matter. Unlike other dark matter candidates (WIMPS,..) it is not an object which has been invented exclusively in order to explain dark matter; this positive energy stuff made its debut already in Wigner's 1939 paper which was written in the same decade in which Zwicky discovered dark matter. What was missing for more than seven decades was an understanding of its inherent noncompact localization in terms of a field theoretic description. Unlike explanations in terms of known matter there are no astrophysical restrictions on the Wigner stuff except those coming from galactic changes of the gravitational balance. This permits to adjust its density to whatever it takes to obtain agreement with the measured gravitational balance over galactic distances. In particular the observational reasons why normal zero mass matter (photons,

[12] The problem of how an e-dependent energy-momentum tensor can be related with the generators of (e-independent) spacetime transformations remains open.

gravitons, massless neutrinos) cannot account for dark matter do not apply to Wigner's stuff; its noncompact nature provides it with the ability to "stick" to galaxies or clusters of galaxies despite its vanishing rest mass.

As mentioned in the introduction there is no other proposal which fulfills Poppers falsification criterion as perfect as Wigner's stuff; any counter-registered event, for which there exist convincing reasons to believe that it is caused by the presence of dark matter, would throw the present proposal of identifying Wigner's stuff with galactic dark matter into the dustbin. It seems somewhat paradoxical that it is the only kind of theoretical positive energy matter whose verification of existence in nature depends on its invisibility with respect to earthly particle counters. There is however the before mentioned problem to understand how such inert noncompact stuff got into our universe in the aftermath of a big bang.

A rough look at the observational situation (about which the author has no astrophysical expertise) with respect to the contribution of massless matter seems to indicate that photons, gravitons and even additional types of massless neutrinos would not be sufficient to account for the necessary gravitationally observed amount of dark matter. With the inclusion of the Wigner stuff the astrophysical observational upper limit restrictions would be eliminated since inert matter is by definition not subject to (non-gravitational) observational restrictions; the density of the stuff can be adjusted so that it fits the amount of gravitationally inferred dark matter. Particle physicists who expected an explanation of dark matter in the present work in terms of yet another kind of WIMPS/-inos will be disappointed. The present proposal is much more fundamental but not necessarily more acceptable. But it is the only attempt which will remain after all efforts to identify earthly WIMPS/-inos or to place the burden on large distance modification of Einstein/Newton gravity have failed. The Wigner stuff is a third proposal to explain dark matter which differs significantly from the two existing ones.

The present proposal for dark matter is not the result of astrophysical expertise, but rather of conceptual curiosity about Wigner's positive energy stuff. Even if astrophysicists will be able to exclude this gravitating but otherwise inert stuff as a contender for galactic dark matter, the historical amazement about its theoretical discovery in the same decade as Zwicky's observation of dark matter and the surprise about the more than 6 decades lasting effort [5] [9] to unravel its possible field theoretic physical properties will still remain. Its theoretical importance for the understanding of the field theoretic description of all three classes of Wigner's positive energy matter and its historic role for discovering the relevance of string-localization of interacting ordinary matter is beyond doubt. Until astrophysical arguments for its exclusion are found, it should be added to the list of dark matter candidates.

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